Financial Crises: Theories and Models
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1 Introduction

Financial crises are infamous for their often disastrous consequences. Moreover, there does not always seem to be a relationship between a ¯nancial crisis and changes in the economic fundamentals, which makes these crises hard to predict. To explain this feature, economists have pointed to the self-ful¯lling nature of ¯nancial crises. In economic theory, a ¯nancial crisis is often viewed as a random shift from one equilibrium to the other. However, this argument is only an informal one. What triggers these self-ful¯lling beliefs, and what should economists advise politicians? These questions cannot be addressed by theories which point only to the random nature of ¯nancial crises. In his lectures, professor Shin introduced a theory and some models that can give more insight in the onset of ¯nancial crises and their probability of occurrence. First, he freshened our knowledge of the more traditional models of ¯nancial crises (section 2) and evaluated them (section 3). In order to improve on these models, professor Shin used the theory of global games (section 4). The rather technical introduction in this game-theoretic concept proved to be worthwhile when professor Shin showed some very interesting applications (sections 5 and 6).

2 First and second generation models

After the break-down of the Bretton Woods system, numerous countries have tried to peg their exchange rate at some ¯xed level or to some other currency's exchange rate. Almost none of these attempts turned out a success; eventually, the peg was given up. In a lot

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of cases, this happened after a speculative attack. In this period, several models evolved that tried to explain these stylized facts. Two important models are discussed below.

2.1 The Krugman (1979)-model

In this model, the central bank has two different tasks. Firstly, it has to defend the exchange rate by holding the monetary base fixed. Its second objective is to monetize (part of) the exogenously given fiscal deficit by buying domestic currency bonds b. Define the change in domestic currency holdings by the central bank $\dot{b}_t = \mu$. Now, in order to keep the monetary base fixed, the central bank has to soak up its foreign currency reserves. Obviously, this policy is not sustainable, for at time T, say, the bank will run out of foreign currency reserves. From that time on, the central bank will no longer be able to keep the exchange rate at the fixed level. So the interesting question in the model is not whether the peg will fail, but rather when this will occur.

Once the central bank runs out of foreign reserves, at time T, the money supply will begin to rise at rate μ. This means that the expected rate of depreciation will jump from zero to μ instantaneously, causing the demand for real money balances to drop sharply. The discrete change in the exchange rate at time T is of course anticipated by speculators, and so they will all switch out of domestic currency into foreign reserves before time T. This explains that in this setting a speculative attack is inevitable. It is obvious, however, that this speculative attack will not occur at time T, precisely because it is anticipated. To see when it will occur, Krugman introduces the concept of the shadow exchange rate, which is the floating exchange rate that would prevail if all the central bank's foreign assets had already passed into private hands. In other words, the speculators, who know that the central bank's policy is not sustainable, have already discounted this knowledge in their shadow exchange rate. In our model this shadow exchange rate rises over time at rate μ. So there is one point where the shadow exchange rate is exactly as high as the fixed exchange rate. Precisely at this point the speculative attack will occur, because this is the only point where the change from a fixed to a floating exchange rate can occur without a discontinuous change. Therefore, this is the only point that is consistent with perfect foresight.

Note that in the Krugman model, perfect foresight is assumed in case of the speculators, whereas the central bank does not seem to care about the long-term sustainability of its policy. There is no a priori reason why the central bank would be myopic. Moreover, speculative attacks typically seem to have a self-fulfilling nature, in that an exchange rate will collapse if enough people expect it to collapse. However, this self-fulfilling nature is not captured by the Krugman model.

2.2 The Obstfeld (1996)-model

The trade-off between two conflicting objectives of the government that was central in the Krugman model is also central in the more general model by Obstfeld. In this model, the monetary authority may for example try to avoid a currency attack by raising the interest rates, but this may have adverse effects on the domestic economy. The pain of sticking to the fixed exchange rate will in general be greater the heavier the attack.

In the Obstfeld model, the conflict between the "domestic targets" and sticking to a fixed (but adjustable) exchange rate is made explicit in a loss function,

$$L = (y - \bar{y})^2 + \chi\pi^2 + c(\pi) \tag{1}$$

where y is output, $\bar{y}$ is the output target, χ is the relative weight the authorities place on inflation stabilization versus employment stabilization, $\pi = \underline{e}$ is the change in exchange rate or inflation and $c(\pi)$ is the cost of changing the currency peg. Furthermore, output is determined by an expectation augmented Phillips curve:

$$y = \tilde{y} + (\pi - \pi^e) - z \tag{2}$$

where $\tilde{y}$ is the natural level of output, π^e is the private sector's belief of π, and z is the noise term. So, deviation of output from its natural rate depends on unexpected changes in the exchange rate and on random shocks. The private sector chooses π^e before observing either z or π, whereas the government chooses π after observing both z and π^e.

The government defends the fixed exchange rate as long as the random shocks are not too large in absolute value. When the shocks are too large, using monetary policy for keeping the exchange rate fixed is too costly, and the government rather uses it to stabilize output. We can see that in this model the relationship between the rational expectation of inflation and the private sector's belief about inflation, π^e, may be very complicated. The problem is that a rise in inflation expectations raises the cost of keeping the exchange rate fixed. This makes it less unattractive for the government to lower the exchange rate indeed, and therefore raises the probability of adjustment of the exchange rate. Since the equilibrium condition is that the rational expectation and the private sector's belief about inflation coincide, there may very well be multiple equilibria in this model. And because of the existence of multiple equilibria, the government may not be able to enforce its most favored equilibrium. A small change in expectations may already cause a shift from one equilibrium to another. Such a jump would look very much like a speculative attack in the Krugman model, forcing the government to abandon the currency peg.

3 The third generation?

The most important contribution of the above models is that they have very clearly pointed out that defending a currency peg entails costs. Therefore, defending cannot go on forever. Whatever the perceived bene¯ts and whatever the public pronouncements, there is a pain threshold at which the costs of defending the peg outweigh the bene¯ts.

Furthermore, the Obstfeld model does shed some light on the self-ful¯lling nature of currency attacks. It also leaves open some important issues, however. A jump from one equilibrium to another can only be explained in an informal manner; a more formal explanation would be preferable. Moreover, the model assumes that the private sector can be represented by a single agent. The assumption does no longer hold if there are pay-o® interactions between the agents, because then not only the agent's own beliefs matter, but also the beliefs of others. This situation seems to be typical for currency attacks, where the optimal action depends on what other agents choose.

This can easily be seen by looking at the many parties that play a role in currency attacks, e.g. speculators, domestic ¯rms, domestic banks, depositors, foreign banks. If one of the parties undertakes a painful action, i.e. an action that makes it more costly to keep the exchange rate ¯xed, it is in the interest of the other parties that they also choose painful actions. Take, for example, the case where a government has pegged its currency to the US dollar. When speculators have information that makes them believe that the domestic currency will devalue, they will borrow the domestic currency and buy dollars. In response to this, domestic ¯rms and banks will also start to sell the domestic currency, because of hedging purposes. Hence, actions which increase the pain are mutually reinforcing. So a realistic model of currency attacks would have to take into account that the beliefs of other players matter.

Another feature of the Obstfeld model is that an attack on a government with weak fundamentals (low costs of adjusting the exchange rate, a high output target, low Â) is more likely to lead to a shift in equilibria. So, given a speculative attack, a government with strong in°ation fundamentals is more likely to resist this attack than one with weak fundamentals. The model does not tell us anything, however, about the relationship between the fundamentals and the private sector's beliefs. In this model, the beliefs can be chosen independently of the fundamentals, while it seems more plausible to assume that the underlying fundamentals have some in°uence on the private sector's belief about in°ation.

4 Global Games

Central in the Morris and Shin (1998) model of currency attacks is the concept of global games, first introduced by Carlsson and Van Damme (1993). It is this theory that enables them to derive a unique equilibrium. Multiple equilibria outcomes often depend crucially on the assumptions of common knowledge of the fundamentals and perfect knowledge of the other agents' actions in equilibrium. But one may argue that common knowledge is too simple a picture. Although market participants do seem to be very well informed because of the availability of a lot of information, this information may still be imperfect. Moreover, because different market participants may use different sources of information, there may be disparities in their information. Therefore, even when everyone knows the fundamentals, they may not know that everyone knows the fundamentals. Still higher orders of uncertainty may be relevant. It is thus unlikely that common knowledge adequately describes the underlying information structure. We will see that changing the assumptions of common knowledge and perfect information solves the multiple equilibrium problem, and results in a model that can address the issues that were unexplained by the first and second generation models.

4.1 Perfect information

Consider first the following perfect information benchmark model. There are two identical investors, each with an endowment of one unit. This unit can either be consumed or invested in a risky project. The output per head from the risky investment is

$$\begin{cases} R & \text{if both invest} \\ bR & \text{if only one invests} \end{cases}$$

where R is a lognormal random variable, and with $0 < b < 1$ and $\log b = -1$. We assume that R is chosen by nature at the beginning of the game, after which it becomes common knowledge, and the players decide wether or not to invest. Note that investing is mutually complementary, in the sense that the return to investment is higher when both invest. Both investors have a logarithmic utility function $u(c) = \log c$, so the utility of consuming the endowment is 0. When we define $r = \log R$, we get the following payoff matrix:

	Invest	Refrain
Invest	$r; r$	$r - 1; 0$
Refrain	$0; r - 1$	$0; 0$

Observe that for $0 < r < 1$ there are two pure equilibria, (Invest, Invest) and (Refrain, Refrain). When we extend the game to more players and mixed equilibria, there can be very many equilibria. So, ex ante, the outcome of this game is indeterminate. However, the Pareto efficient outcome is that both players invest whenever $r \geq 0$.

4.2 Imperfect information

Now we change the information structure of the game. The return on investment, R, is no longer common knowledge after it is chosen by nature. Rather, there is uncertainty about the returns. Instead of knowing the return exactly, the players receive noisy signals about R. Based on these signals, the players decide whether to invest or to refrain. r is normally distributed with mean $\bar{r}$ and variance $1/\alpha$: $r \sim N(\bar{r}; 1/\alpha)$. $\bar{r}$ is assumed to be common knowledge. A larger α thus reflects less uncertainty about the return to investment. So r can be viewed as reflecting the state of nature. Investor i receives a noisy signal x_i of this state of nature, $x_i = r + \varepsilon_i$, with $\varepsilon_i \sim N(0; 1/\beta)$ and the ε_i are independent. When β gets large, the signal becomes more precise.

Based on the signal x_i, player i forms an estimate of r, ρ_i:

$$\rho_i = E(r \| x_i) = \frac{\alpha\bar{r} + \beta x_i}{\alpha + \beta} \tag{3}$$

This posterior belief about r is player i's best estimate of r. Given the prior mean of r, there is a one to one correspondence between ρ_i and x_i. The expected payoff for investor i conditional on x_i is $E(r - I|\rho_i)$, where I is an indicator function for the other player's investment, which has the following form:

$$I = \begin{cases} 1 & \text{if opponent refrains} \\ 0 & \text{if opponent invests} \end{cases}$$

From the definition of the posterior belief, it follows that the expected conditional payoff can be rewritten as

$$E(r - I|\rho_i) = \rho_i - E(I|\rho_i) \tag{4}$$

So player i's expected payoff depends on his posterior belief and on his expectation about the action of his opponent conditional on his posterior belief. A strategy for player i is a function $\rho_i \mapsto \{\text{Refrain}, \text{Invest}\}$. When we think of the signal x_i that player i receives as his type, we can solve this game for the Bayesian Nash equilibria. The equilibrium strategies have the form of a pair of strategies such that the investor's expected payoff is maximized, conditional on his posterior belief ρ_i, and given the strategy of the other investor.

4.2.1 Switching strategies

We will start solving this game by concentrating on switching strategies. So we want to find an equilibrium that consists of strategies of the following form

$$\begin{cases} \text{Invest if } \rho_i \geq \rho^* \\ \text{Refrain if } \rho_i < \rho^* \end{cases} \tag{5}$$

According to this strategy, player i should invest if his posterior belief about r is high enough, and refrain otherwise. In order for this strategy to be an equilibrium strategy, player i must be indifferent between investing and refraining when his posterior belief is at the switching point ρ^*. In other words, the expected conditional payoff of investing, $E(r - I|\rho^*)$, must equal that of refraining, 0. From (4), we know that $E(r - I|\rho^*) = \rho^* - E(I|\rho^*)$. The indicator function I takes the value 1 if and only if $\rho_j < \rho^*$, and therefore $E(I|\rho^*) = \Pr(\rho_j < \rho^*|\rho^*)$. Player j's posterior belief will be below the switching point if and only if

$$\frac{\alpha \bar{r} + \beta x_j}{\alpha + \beta} < \rho^* \tag{6}$$

We can rewrite this as $x_j < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})$. Based on ρ^*, player i can infer player j's signal x_j. Conditional on ρ^*, r is normally distributed with mean ρ^* and variance $\frac{1}{\alpha+\beta}$ (because of the independence of the error terms). Therefore,

$$x_j|\rho^* \sim N\left(\rho^*; \frac{1}{\alpha + \beta} + \frac{1}{\beta}\right) = N\left(\rho^*; \frac{\alpha + 2\beta}{\beta(\alpha + \beta)}\right) \tag{7}$$

Now we can calculate the probability,

$$\begin{aligned} \Pr(\rho_j < \rho^*|\rho^*) &= \Pr\left(x_j < \rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r})\,|\rho^*\right) \\ &= \Phi\left(\sqrt{\frac{\beta(\alpha+\beta)}{\alpha + 2\beta}}\left(\rho^* + \frac{\alpha}{\beta}(\rho^* - \bar{r}) - \rho^*\right)\right) \\ &= \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \end{aligned} \tag{8}$$

where Φ is the cumulative normal distribution centered on the ex ante mean $\bar{r}$, and

$$\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)} \tag{9}$$

Since we know that in equilibrium $\rho^* - E(I|\rho^*) = 0$, we can conclude that

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \tag{10}$$

It is straightforward to see that the expected payoff is increasing in ρ_i, and therefore it is optimal for player i to adopt the switching strategy around ρ^*. Both players playing these strategies is therefore a Bayesian Nash equilibrium.

4.2.2 Unique equilibrium

In the last section, we concentrated on switching strategies. We will now see that the switching strategy is indeed a unique equilibrium, provided that the noise is small enough. The proof is informal; see Morris and Shin (2001) for a more formal proof.

The first condition for a unique equilibrium is that there is a unique switching strategy, i.e., there is only one solution to (10). This implies that the graph of the scaled cumulative normal should cross the 45°-line only once. Since the slope of the cumulative normal reaches a maximum at the mean, $\bar{r}$, a sufficient condition for the existence of at most one switching strategy equilibrium is that the slope of $\Phi\left(\sqrt{\gamma}(\rho^* - \bar{r})\right)$ is smaller than or equal to one at $\bar{r}$. So,

$$\sqrt{\gamma} \cdot \phi\left(\sqrt{\gamma}(\rho^* - \bar{r})\right) \leq 1 \tag{11}$$

Evaluating at $\bar{r}$ gives $\gamma \leq 2\pi$; if γ is small enough, the switching strategy equilibrium is unique. From the definition of γ we can see that when the signal about the state of nature is very precise relative to the uncertainty about the return on investment, that is, when β is large relative to α, γ will get small. So when the noise is small enough, there is a unique switching strategy equilibrium.

It remains to be shown that, provided that there is a unique equilibrium in switching strategies, there are no other equilibrium strategies than switching strategies. This can be done by iterated deletion of dominated strategies. Note that if r is negative, it is never optimal for player i to invest, no matter what the other player does. Therefore, if ρ_i is sufficiently low, refraining from investment is a dominant action. Thus, both investors rule out strategies which invest for signals lower than $\underline{\rho}_1$, the threshold value, and they also rule out strategies of each other which invest for signals lower than $\underline{\rho}_1$. Now, the most "optimistic" strategy that is left is a switching strategy around $\underline{\rho}_1$, i.e., a strategy that always invests for $\rho_i > \underline{\rho}_1$. The best response to this strategy is to play a switching strategy around $\underline{\rho}_2$, where $\underline{\rho}_2$ solves

$$u\left(\underline{\rho}_2; \underline{\rho}_1\right) = 0 \tag{12}$$

where $u(\rho; \rho^*)$ is the expected utility from investing conditional on one's posterior belief ρ when the other investor follows a switching strategy around ρ^*. To see that this is a best response, note that the expected return on investment is increasing in the investor's own posterior belief. But then, investment cannot be optimal for a signal below $\underline{\rho}_2$. Furthermore, no player will believe that the other player will play the switching strategy around $\underline{\rho}_1$, but rather a more cautious one. The incidence of investment by the other player will thus be lower than that implied by the switching strategy around $\underline{\rho}_1$. So every strategy that invests for $\rho_i < \underline{\rho}_2$ is dominated. Since $u(\cdot;\cdot)$ is increasing in its first argument and decreasing in its second argument, i.e., the posterior belief of the other player, $\underline{\rho}_2 > \underline{\rho}_1$. So the second round of deletion narrows the possible equilibrium strategies down to strategies that invest only if $\rho_i > \underline{\rho}_2$. Through iterated dominance, any strategy that invests $\rho_i < \underline{\rho}$ is deleted, where $\underline{\rho}$ is the smallest solution to the equation $u(\rho;\rho) = 0$. A similar argument can be derived for sufficiently high values of ρ_i. Again, through iterated dominance any strategy is removed that refrains for $\rho_i > \bar{\rho}$, where $\bar{\rho}$ is the largest solution to the equation $u(\rho;\rho) = 0$. But since there is only one solution to the equation $u(\rho;\rho) = 0$, the smallest and the largest solution are the same and there remains precisely one strategy after iteratively eliminating all dominated strategies.

Note that this is a very strong result. As we saw in section 4.1, in the full information case, there are multiple equilibria. Here we see that introducing uncertainty about the state of nature and about the beliefs of the other investor leads to a unique equilibrium. This result is independent of the degree of uncertainty; even when the signal about r becomes infinitely precise, there is a unique equilibrium.

We also saw in section 4.1 that the Pareto efficient outcome is that both players play a switching strategy around $\rho_i = 0$. In the unique switching strategy equilibrium that we analyzed, ρ^* is not equal to zero. Indeed, we can think of ρ^* as a measure of inefficiency; the higher ρ^*, the more inefficiency is introduced due to the coordination problem.

5 Bank Runs

The concept of global games can be applied to many economic situations. In this section, we show an application to bank runs. We first analyze a simple model of bank runs, in the spirit of Diamond and Dybvig (1983). Then we show how the concept of global games can be used in this application.

5.1 The Model

There are three dates, $\{1, 2, 3\}$, and a continuum of consumers. Every consumer is endowed with one unit of the consumption good. Consumption takes place either at date 1 or at date 2. There is measure λ of impatient types who have utility function $u(c_1 + c_2) = \log c_1$ and a measure 1 of patient types who have utility function $u(c_1 + c_2) = \log(c_1 + c_2)$, where c_1 and c_2 denote consumption at date 1 and date 2 respectively. Since the consumers learn of their types only at date 1, the ex ante probability of being an impatient type is $\frac{\lambda}{1+\lambda}$

The consumers can either store the consumption good for consumption at a later date, or they can deposit it in the bank. At date 1, after learning of their types, the consumers who deposited their wealth in the bank must decide wether they will leave their money in the bank or will withdraw the sum that is permitted in the deposit contract.

The bank's portfolio consists of two assets: it may either hold the deposited wealth in cash, which gives a rate of return of 1, or he may invest it in a risky project, with rate of return $R \cdot e^{-l}$ obtainable at date 2. We assume that $0 < \log R < 1$. l is the proportion of the resources invested that is withdrawn at date 1. Early withdrawal is thus costly, in that it reduces the rate of return, i.e., the project is illiquid. When we define $r = \log R$, the rate of return can be written as e^{r-l}. It is assumed that only the bank has access to the investment project.

5.2 Optimal contract under perfect information

A contract is a pair $(c_1; c_2)$, i.e., a promise by the bank to pay c_1 at date 1 if the consumer withdraws and c_2 at date 2 if the consumer does not withdraw. The optimal contract maximizes expected utility

$$\frac{\lambda}{1+\lambda} u(c_1) + \frac{1}{1+\lambda} u(c_2) \tag{13}$$

subject to two constraints:

$$\lambda c_1 + \frac{c_2}{R} \leq 1 + \lambda \tag{14}$$

$$u(c_1) \leq u(c_2) \tag{15}$$

The first constraint is the budget constraint for the bank: on the left-hand side are the expected costs of the contract $(c_1; c_2)$, which must be less than or equal to the total amount of deposits. The second constraint is the incentive compatibility constraint, which requires that patient types will indeed choose to leave their money in the bank. From the budget constraint, it is straightforward to see that $c_1 = 1$ and $c_2 = R$ is a binding solution. For this solution,

$$u(c_1) = 0 < r = u(c_2) \tag{16}$$

so that the incentive constraint is satisfied. In the optimal contract, any depositor can withdraw the whole of their 1 unit deposit at date 1. At worst, a consumer gets his money back on date 1. Therefore, depositing in the bank is a weakly dominant action for every consumer, and, at date 0, all consumers deposit their money in the bank.

Unfortunately, the optimal contract gives rise to multiple equilibria at date 1. At date 1, all the impatient types withdraw their money, so they conform to the optimal contract.

For the patient types, the story is more complicated, however. If a patient consumer expects all other patient consumers not to withdraw, he receives $r > 0$ at date 2 if he conforms to the contract. But if a patient consumer expects all other patient consumers to withdraw ($i.e., l = 1$), then his utility from not withdrawing is $r - 1 < 0$. So in this case it is optimal for the patient consumer to withdraw. So there are two equilibria: one where everyone conforms to the optimal contract and one where everyone withdraws at date 1. Note the similarity between the outcome of this simple model of bank runs and the outcome of the coordination game in section 4.1. As in section 4.1, there are multiple equilibria, with desirable and undesirable outcomes, and it is impossible to tell which one will prevail. Therefore, the nature of a bank crisis is completely random and there is no correlation between a bank crisis and the underlying state r.

5.3 Imperfect information

Now we change the information structure of the above model and introduce uncertainty about the log return parameter r. Suppose that r is a random normal variable with mean $\bar{r}$, and variance $1/\alpha$. Furthermore, $0 < \bar{r} < 1$. Now the consumers no longer have perfect information about r, but they rather receive a noisy signal $x_i = r + \varepsilon_i$, with ε_i normally distributed with mean 0 and variance $1/\beta$, and independent across consumers. It is typically assumed that $\beta \ll \alpha$, which means that there is relatively little noise in the signal x_i[1].

What does an equilibrium look like in this game? At date 0, the fundamentals of the game are known, and nature chooses r. At date 1, the depositor learns of his type and receives a signal x_i about r. Conditional on this signal, he forms his belief ρ about the true state r, and about the beliefs of the other depositors. Based on this information, he decides whether or not to withdraw. So a strategy for a depositor is a function that maps from the realization of the signal to the action set {Withdraw, Not withdraw}. A profile of strategies is an equilibrium if, conditional on the information available to depositor i and given the strategies followed by other depositors, the action prescribed by i's strategy maximizes his expected utility. Observing that the structure of the game is exactly the same as the coordination game described in section 4.2, it is straightforward to show that the unique equilibrium of this game is that everyone plays a switching strategy around ρ^*, which in this context means that everyone withdraws when $\rho < \rho^*$, where

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \tag{17}$$

[1] Remember from section 4 that this guarantees a unique switching strategy equilibrium.

Each individual depositor will withdraw whenever his posterior belief, which is based on the realization of his signal, is below the threshold value ρ^*, or whenever

$$\frac{\alpha \bar{r} + \beta x_i}{\alpha + \beta} < \rho^* \qquad (18)$$

This means that a depositor will withdraw whenever the signal he receives falls below the critical value

$$x^*(\rho^*; \bar{r}) \equiv \frac{\alpha + \beta}{\alpha} \rho^* - \frac{\alpha}{\beta} \bar{r} \qquad (19)$$

Since $x_i = r + \varepsilon_i$, the proportion of depositors that withdraw is a function of r, and is given by

$$l(r) = \Phi\left(\sqrt{\beta}\,(x^*(\rho^*; \bar{r}) - r)\right) \qquad (20)$$

This can easily be seen by recognizing that $l(r)$ is the proportion of the depositors whose posterior belief is below ρ^*.

5.4 Implications

According to the Diamond and Dybvig model, the incidence of withdrawal is uncorrelated with fundamental values like the return r, and is instead completely random. However, empirical studies suggest that there are correlations between fundamentals and bank runs. This evidence should not lead to a full rejection of the Diamond-Dybvig story, because it sheds light on the coordination problem that seems to play a crucial role in the onset of bank runs. The above analysis provides a third way of looking at bank crises. As can be seen from (20), the fundamentals do matter. l is decreasing in both r and ρ^*, which means that a higher rate of return does reduce the incidence of withdrawal. But fundamentals are definitely not the whole story. The banking panic is self-fulfilling in the sense that individual investors only withdraw because they expect others to do so. Thus, the theory suggests both that bank runs are correlated with poor fundamentals and that inefficient self-fulfilling panics occur.

6 Creditor Coordination and the Price of Debt

Creditors face a coordination problem when facing a borrower in financial distress. A creditor may be tempted to foreclose on the loan or seize any assets it can, fearing similar actions by other creditors. This fear is self-fulfilling, in that the foreclosure of loans or the liquidation of assets increase the probability that the project fails. Through this coordination problem, viable projects may fail. Therefore, this coordination problem will have an e®ect on asset pricing. However, most theories of asset pricing do not explicitly deal with coordination problems. One obvious reason for this is that coordination problems lead to multiple equilibria, as we saw above.

There is also empirical evidence supporting the importance of coordination problems. For example, in the ¯nancial market turbulence of 1998, which was a period of extremely high yield spreads, access to bank lending was much less a®ected than access to the bond market. The solution to this empirical puzzle may be that bank lending, with a small number of creditors, su®ered much less from the coordination failure between creditors than lending through the bond market. There is also empirical evidence suggesting that the classical way of valuing debt, by using the Merton (1974) model, yields predictions of corporate bond prices which are higher than the actual observed prices, and that the prediction error is higher for riskier bonds.

In order to incorporate the coordination problem into a theory of asset pricing, we need to know the incidence of coordination failure as a function of the fundamental values underlying the asset. Once we know this, we can in corporate the risk of coordination failure into the price of the debt.

6.1 The Model

There is a group of creditors, indexed by the unit interval $[0;1]$, ¯nancing a project with an uncertain payo® V, which is realized at the ¯nal stage. Each individual creditor is small, i.e., the individual stake is negligible as a proportion of the whole. The face value of repayment at the ¯nal stage equals 1. At an interim stage, the investors have an opportunity to review their investment. They can choose either to roll over the loan until maturity or to foreclose at the interim stage. Foreclosure gives a payo® ¸, $0 <$ ¸ < 1, which can be thought of as the value of selling collateral. The payo® to rolling over depends on whether the project will succeed or not. This is in°uenced by two factors: the underlying state µ and the proportion l of creditors who foreclose. The realized value of the project is given by

$$V(\mu; l) = \begin{cases} 1 \text{ if } zl \cdot \mu \\ 0 \text{ if } zl > \mu \end{cases}, \tag{21}$$

where $z > 0$ is a parameter which measures the severity of the disruption caused by foreclosure. Suppose the creditors have perfect certainty about µ. Note that if $\mu > z$ the project will succeed, and the creditors will always choose to roll over the debt, irrespective of what the other creditors do, i.e., irrespective of l. Conversely, if $\mu < 0$, it is always better to foreclose at the interim stage. However, when µ 2 $(0; z)$, whether rolling over or foreclosure is optimal depends on l, that is, on what the other creditors choose. Therefore, there is a coordination problem when $\theta \in (0; z)$. Note that this coordination problem is akin to the bank run problem described in section 5.2. So, again, we end up with multiple equilibria.

6.2 Imperfect Information

As in the bank runs model, we proceed by assuming imperfect information. At the beginning of the game, the creditors know that θ is a normally distributed random variable with mean y and variance $1/\alpha$. At the interim stage, the creditors receives information concerning θ, in the form of the realization of a noisy signal $x_i = \theta + \varepsilon_i$, where the ε_i's are independent and normally distributed with mean 0 and variance $1/\beta$. Based on this signal, creditor i's posterior belief about θ is normally distributed with mean

$$\xi_i = \frac{\alpha y + \beta x_i}{\alpha + \beta} \tag{22}$$

and variance $1/(\alpha + \beta)$. Now, every creditor adopts a strategy, which tells him whether to roll over or to foreclose, conditional on his posterior. We want to find an equilibrium, that is a strategy for each creditor, such that following this strategy maximizes a creditor's payoff conditional on his signal and given that the other creditors play the strategies in the profile. Analogous to the analysis of section 4, it can be shown that there is a unique equilibrium, provided that the noise is small enough.

First, we define $U(\xi)$, the payoff to rolling over given ξ, if all other creditors follow a switching strategy around ξ. Note that there is an equilibrium in switching strategies around ξ if ξ solves $U(\xi) = \lambda$, that is, the expected conditional payoffs of rolling over and foreclosure are equal. Furthermore, using the iterated dominance argument that we used in earlier sections, we can show that if there is a unique switching strategy equilibrium, then there is no other equilibrium. For a switching strategy to be optimal, the payoff of rolling over must be increasing in ξ for all ξ, $U'(\xi) \geq 0$. Note that this is also a sufficient condition for uniqueness of the switching strategy equilibrium. It can be shown that $U'(\xi) \geq 0$ for all ξ if and only if[2] $\alpha/\sqrt{\beta} \leq \sqrt{2\pi}/z$. So when this condition is met,

[2] Observe that

$$U(\xi) = \int_{\psi}^{\infty} \phi\left(\sqrt{\alpha+\beta}\,(\theta - \xi)\right) d\theta = 1 - \Phi\left(\sqrt{\alpha+\beta}\,(\psi - \xi)\right),$$

where $\phi(.)$ is the density of the standard normal and ψ is the failure point.

$$U'(\xi) = -\sqrt{\alpha+\beta} \cdot \phi\left(\sqrt{\alpha+\beta}\,(\psi - \xi)\right) \cdot \left(\frac{\partial \psi}{\partial \xi} - 1\right)$$

there is a unique equilibrium.

Now we want to solve for this unique equilibrium. If l is determined by everyone following the switching strategy around ξ, what is the critical value of θ for which the project succeeds? We call this critical value of θ the failure point. Note that the failure point depends on the switching point ξ, and that the switching point depends on the failure point. The failure point is the point that solves

$$\theta = zl \tag{23}$$

Conditional on state θ, x is normally distributed with mean θ and variance $1/\beta$. Therefore, if all creditors follow the switching strategy around ξ, the proportion of the creditors who have a signal that is lower than the critical value is

$$l = \Phi\left(\sqrt{\beta}\,(x(\xi; y) - \theta)\right) \tag{24}$$

Substituting this into (23) gives an expression for the critical value of θ at which the project succeeds, the failure point ψ:

$$\psi = z\Phi\left(\sqrt{\beta}\,(x(\xi; y) - \psi)\right) \tag{25}$$

We also know that $U(\xi) = \lambda$, which we can rewrite[3] as

$$1 - \Phi\left(\sqrt{\alpha + \beta}\,(\theta - \xi)\right) = \lambda \tag{26}$$

From the last two equations we can solve for the failure point

$$\psi = z\Phi\left(\frac{\alpha}{\sqrt{\beta}}\left(\psi - y + \Phi^{-1}(\lambda)\frac{\sqrt{\alpha + \beta}}{\alpha}\right)\right) \tag{27}$$

6.3 Implications

Note that $\psi > 0$. Since the efficient outcome is $\psi = 0$, there is an efficiency loss due to the coordination problem. We can also see that the failure point depends on the parameters of the problem. One important conclusion is that failure occurs at higher values of the fundamentals when y, the mean of θ, is low, that is, when the debt is of low quality.

To see the consequences of this model, let's look at the value of an unsecured loan. The owner of such an asset only receives a positive payoff when the true state is higher than the failure point ψ. The ex ante price of such a loan given y is

$$W(y) = \int_{\psi}^{1} \phi\left(\sqrt{\alpha}(\theta - y)\right) d\theta = 1 - \Phi\left(\sqrt{\alpha}(\psi - y)\right) \tag{28}$$

Hence, $U'(\xi) \geq 0$ iff $\frac{\partial \psi}{\partial \xi} \leq 1$. Totally differentiating (25) gives the result.

[3]See footnote 2.

The e®ect of a shift in the ex ante mean y is thus given by

$$\frac{dW}{dy} = p_{\overline{®} ¢ Á} ¡ \frac{@Ã}{@y} p_{\overline{®} ¢ Á} \qquad (29)$$

The ¯rst term is the "conventional" e®ect: if y decreases, the distribution shifts to the left, and, with Ã ¯xed, this increases the left-hand tail. So a decrease in the fundamental value increases the downside risk of an asset. However, the second term is the novel feature. It arises from the fact that a shift in y also causes Ã to shift. For high quality loans, i.e., for high y, lower realizations of the signal will still lead to a successful outcome of the project, that is, it will lower the failure point. Since @Ã=@y < 0, the second e®ect reinforces the ¯rst e®ect. This second e®ect is due to the coordination problem.

For the creditor, a deterioration in the fundamental values in terms of a fall in y implies that the asset value of the loan is falling at a rate more than proportional to the fall in y. So by neglecting the coordination e®ect, the creditor is overestimating the value of his asset or of his portfolio. This theory thus can explain the evidence against the predictions of the Merton model. The Merton model takes into account only the ¯rst e®ect, and will therefore underestimate the downside risk of an asset and predict too high prices. Furthermore, we observed that this prediction error is increasing in the riskiness of the asset. This feature is also consistent with this theory, because it predicts a higher failure point for lower quality assets, i.e., for lower y. This means that for lower quality debt, the coordination problem leads to more ine±ciency and thus to a lower price.

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