Pricing
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1 Introduction

The lectures of professor McAfee introduced us to a range of topics in pricing. He started his series of lectures with some highly interesting examples, indicating the practical relevance of the topics. On the one hand, the theories that professor McAfee introduced us to explained why it may be perfectly rational for a manufacturer to deliberately produce a version of his product that is clearly inferior to the original, and sell it at a lower price. On the other hand, the theories also proved very useful in many (industrial) policy issues related to the (supposed) abuse of monopoly power. In the following, I give a brief review of the most important issues that were dealt with. These issues are price discrimination, quantity discounts, quality premia, peak-load pricing, priority pricing, price dispersion, and durable good pricing.

2 Price discrimination

Examples abound in which the same economic good is sold at different prices to different consumers. In general, this phenomenon, known as price discrimination, can be seen as an attempt by a monopolist to appropriate a larger part of the economic surplus.

Assume there is a single good, with downward sloping demand $p(q)$, and a profit maximizing monopolist. Costs are normalized to zero. In case of a uniform price, the monopolist earns $q_0 p(q_0)$. A two price discriminating monopolist earns $q_1 p(q_1) + (q_2 - q_1)p(q_2)$, where the q's are the optimal quantities. Note that, in this case, welfare can only be increased by selling a higher quantity, as this reduces the inefficiency due to the

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monopolist's quantity rationing. It can easily be shown (see Varian, 1985) that welfare is higher under price discrimination. The intuition is as follows. In the non-discriminatory case, in determining the optimal quantity the monopolist maximizes over all consumers' willingnesses to pay. When discriminating, the monopolist sells q_1 to the consumers with a high willingness to pay. In determining how much to sell to the low-valuation consumers, the monopolist faces a new optimization problem, where the highest valuation consumers can be ignored. This will lead to a total quantity sold of $q_2 > q_0$, thus increasing welfare.

This is not a general result, however. If a monopolist faces n markets, with demand in market i given by $x_i(\mathsf{p})$, where p is the price vector, price discrimination may or may not increase welfare. Note that, in this case, demand is interdependent between different markets. Moreover, it is assumed that the cross-price elasticities are symmetric, i.e. $dx_i/dp_j = dx_j/dp_i$. Simple optimization leads to a generalization of the inverse elasticity rule: $L = -E^{-1}\mathsf{1}$, where L is the n-vector of Lerner indices $(p_i - mc)/p_i$, and E contains the own price elasticities (on the diagonal) and the cross-price elasticities (off the diagonal). So, the price/cost margins are determined by the inverse of the matrix of elasticities.

Charging different prices p_i has two effects. The first effect is that output is reallocated across markets. This causes the marginal rate of substitution to differ among consumers. Since a given quantity is optimally distributed by charging a uniform price, this has a negative impact on welfare. The second effect is a change in output; a higher output reduces the monopoly pricing distortion, thereby increasing total welfare. Thus, price discrimination may or may not lead to higher output. For example, output may be increased by serving markets that were not served under uniform pricing because the price was too high. The two effects imply that a necessary condition for price discrimination to increase welfare is that it increases total output.

2.1 Ramsey pricing

Consider a social planner whose objective is to maximize total welfare subject to a break-even constraint for the monopolist. Which prices would be set by the social planner and how does this differ from the prices that would be set by a monopolist? The planner faces the following problem:

$$\max_{p_i} u(\mathsf{x}) - c(\mathsf{x}) \text{ s.t. } \mathsf{px} - c(\mathsf{x}) \geq \pi_0$$

Writing the Lagrangian and solving for the first order condition gives the Ramsey price solution

$$-\frac{\lambda}{1+\lambda} = \sum_{j=1} \frac{p_j - mc}{p_j}\epsilon_{ij},$$

where λ, the Lagrange multiplier is the marginal increase in welfare associated with a decrease in firm profits, and ϵ_{ij} is the cross-price elasticity of substitution

$$\epsilon_{ij} = \frac{p_j}{x_i}\frac{dx_i}{dp_j}$$

Note the similarity with the optimizing monopolist outcome. The social planner would also set prices according to some inverse elasticity rule. So, whether or not the monopoly prices will be implemented depends on λ. If $\lambda \to \infty$, the Ramsey price converges to the monopoly outcome, whereas $\lambda = 0$ gives a markup of 0, i.e. marginal cost pricing. The Ramsey solution is thus in between marginal cost pricing and monopoly pricing, depending on λ.

2.2 Arbitrage

The interdependencies of demand depend crucially on the possibility of arbitrage. In a situation where there is no possibility whatsoever for arbitrage, the cross-price elasticity will be zero at all prices. This means that the matrix of elasticities E becomes diagonal. In this case, the inverse elasticity rule reduces to:

$$\frac{p_i - mc}{p_i} = -\frac{1}{\epsilon_{ii}}$$

This extreme case of no arbitrage shows that the degree of arbitrage only effects the matrix of elasticities, but does not alter the inverse elasticity and welfare results. If arbitrage is possible, then markets are no longer independent. Note that without arbitrage, the monopolist has a higher markup. It is therefore in his interest to prevent arbitrage.

3 Quantity discounts and quality premia

In general, a monopolist charges different prices in order to appropriate as much consumer surplus as possible. In order to be able to do this, he must know the different consumer willingnesses to pay. But what if willingness to pay is consumers' private information? Clearly, the monopolist especially would like the high valuation consumers to reveal their types, whereas precisely the high valuation consumers are the least willing to give up their private information. What should an optimizing monopolist do in this situation?

3.1 Quantity discounts

Assume there exist two types of consumers; L, who values quantity q at $V_L(q)$, and H, who values quantity q at $V_H(q)$. Furthermore, $V_L(0) = V_H(0) = 0$, which means that both value nothing at zero, and $V'_H(q) \geq V'_L(q)\ \forall\ q > 0$, which means that the high type is always wiling to pay more for an increase in quantity than the low type.

In the case of a quantity discount, different quantities are offered at different per unit prices. The monopolist offers two bundles (q_L, p_L) and (q_H, p_H), with p_i being the price for the quantity q_i, such that both types of consumers are willing to purchase, i.e., $V_i(q_i) - p_i \geq 0$, $i = L, H$, and such that both types choose the bundle that is targeted for their type, i.e., the bundles must be incentive compatible. This means that each type maximizes its utility by choosing the 'right' bundle, or

$$V_i(q_i) - p_i \geq V_i(q_j) - p_j,\ i = L, H,\ j \neq i$$

The monopolist is assumed to have constant marginal cost c, and to maximize profit $\Pi = p_L + p_H - c(q_L + q_H)$

The fact that the high type is always willing to pay more for a given bundle than the low type implies that the monopolist will always choose $q_L \leq q_H$. Otherwise, the high type would prefer the low type bundle, given the latter's lower valuation. The low type bundle will be such that the low type is just willing to purchase, i.e., $V_L(q_L) = p_L$. If his surplus would be strictly positive, the monopolist could raise the prices charged to both types with the same amount, thereby increasing his profit, without violating any constraint. If the low type is willing to buy, i.e., $V_L(q_L) \geq p_L$, and if the high type bundle is incentive compatible, i.e., $V_H(q_H) - p_H \geq V_H(q_L) - p_L$, then it follows that the high type is also willing to buy, for he values quantity higher. Furthermore, the high type bundle will be such that the high type's incentive constraint holds with equality. Otherwise, the monopolist could increase profits by charging a higher p_H.

This leads to the following conclusions:

1. The high type is offered the efficient quantity (i.e., $V'_H(q_H) = c$), whereas the low type gets strictly less than the efficient quantity.

2. The high type has a positive consumer surplus (if $q_L > 0$), whereas the low type gets zero consumer surplus.

The intuition behind this result is as follows. In trying to extract the high surplus of the high type, the monopolist faces the risk that the high type will choose the low type bundle. In order to prevent this from happening, the monopolist offers a relatively low quantity to the low type. Because the high type values quantity more than the low type, this will reduce the risk that the high type chooses the low type bundle. From the monopolist's point of view, the surplus extracted from the low type decreases, but this is more than offset by the increase in surplus extracted from the high type. [1]

Quantity discounts can thus be viewed as price discriminatory actions. By offering different quantities at different per unit prices, a monopolist tries to sell more to consumers who value quantity higher, and extract a larger part of consumer surplus.

3.2 Quality premia

The above analysis can also be interpreted as a model for quality premia. In this case, the monopolist offers a range of qualities to consumers who differ in their valuation of quality. Again, the monopolist offers the efficient quality to the high type and a less than efficient quality to the low type. The objective of this is to deter the high type from choosing the low quality.

Thus, by offering different qualities of the same good at different prices, a monopolist can segment the market. For instance, the different classes offered by railroads and airways can be explained by this model. But also more extreme applications can be seen in practice. It is often observed that a producer offers different qualities, even if the cost of offering the low quality is higher than the cost of offering the high quality. For instance, once a high quality product is produced, a portion of it is damaged or otherwise changed into an inferior product. Even if making the product inferior is costly, it may be optimal for a producer to pay this extra cost. As before, by doing this, he deters the high type to choose the low quality, and is able to appropriate extra surplus by serving the low valuation consumers.

3.3 Tie-ins

Tie-ins arise whenever a manufacturer requires the purchase of one product in order to purchase another product. A bank offering an indivisible array of services, a tour operator selling comprehensive vacation plans, and selling new cars with tires already included, are all examples of tie-ins. Tie-ins, or commodity bundling, can also be analyzed using the model introduced in this section. To see the similarity, consider a quantity discount. The

[1] The analysis can easily be extended to the case where there is a continuum of types with a known distribution.

price of two units of a good is less than twice the price of one unit. In a sense, the two units are bundled, since buying the two units separately involves additional costs. In the case of a tie-in, a producer offers a bundle of two different products together at a lower price than the sum of the individual prices. So, one can analyze the tying of units of several goods in a similar way as that of several units of the same good.

4 Peak-load pricing and Priority pricing

Several issues arise with respect to pricing if capacity is somehow limited. Two issues are peak-load pricing and priority pricing.

4.1 Peak-load pricing

The problem of peak-load pricing arises if a firm has two kinds of costs; capacity costs and a marginal production cost. The idea can be applied to many industries where building capacity is necessary to provide the product or service. Examples are pipelines, airlines, telephone networks, and electricity. In these industries, marginal cost pricing is not sustainable, since it does not take into account the price of capacity. Therefore, a capacity charge is necessary. Assuming that the capacity lasts for more than one period, peak-load pricing deals with the question to what period the charge should be allocated.

Consider a two period problem and let the subscript denote the different periods. The firm's profits are given by

$$\Pi = p_1 q_1 + p_2 q_2 - \beta \max[q_1, q_2] - mc(q_1 + q_2),$$

where β denotes the capacity charge. The problem is analyzed from the viewpoint of a social planner, [2] who will maximize

$$W = \int_0^{q_1} p_1(x)dx + \int_0^{q_2} p_2(x)dx - \beta \max[q_1, q_2] - mc(q_1 + q_2)$$

subject to a profit condition for the firm. Writing the Lagrangian and taking first order conditions gives

$$\frac{p_i(q_i) - \beta 1_{q_i \geq q_j} - mc}{p_i} = \frac{\lambda}{1+\lambda}\frac{1}{\epsilon_i} \; i = 1, 2, \; i \neq j$$

where $1_{q_i \geq q_j}$ is the characteristic function of the event $q_i \geq q_j$. If we let the demand in period 1 exceed the demand in period 2, two cases can be distinguished.

[2] It is assumed that demands are independent. This is not a necessary assumption; it simplifies the analysis, however.

- $q_1 > q_2$
  It follows directly from the first order conditions that all the capacity charge is allocated to period 1. With the capacity charge, the quantity in period 1 still exceeds the quantity in period 2. So, charging a higher price in the peak-load period does not change the peak. Moreover, note that it is socially optimal to charge a higher price in the peak-load period.

- $q_1 = q_2$
  Since the first derivative of $\beta \max[q_1, q_2]$ could be anything between 0 and β, the first order conditions become inequalities. Solving them for $q_1 = q_2 = q$ gives

  $$p_1(q) - mc + p_2(q) - mc = \beta$$

  This equation shows that the capacity charge is shared across the two periods proportional to the inverse demand, $p(q)$. Note that the charge is not shared proportional to elasticities, as was the case in previous applications.

4.2 Priority pricing

An issue that is also related to capacity, is the issue of priority pricing. Suppose that supply is stochastic, and that it is impossible to use prices to ration the market *ex post*, i.e. after the supply has realized. Priority pricing is a means of contracting in advance when capacity is stochastic.

There is a continuum of consumers, each desiring one unit of consumption. The consumers are ranked by their valuations for the good, so that the q^{th} consumer has a value $p(q)$ for the good, and p is downward sloping. Capacity is a random variable with distribution F. Now, priority pricing is a charge schedule c which provides a unit to a consumer paying $c(q)$ whenever realized capacity is q or greater. High valuation consumers, i.e. low q consumers, are willing to pay more to increase the probability that they will be provided a unit of the good.

The schedule has to be designed such that each type of consumer pays the charge targeted to him, i.e., it has to be incentive compatible. In other words, a consumer of type q should choose to pay $c(q)$ for the q^{th} spot in the priority list. So,

$$u(q) = (p(q) - c(q))\,(1 - F(q)) \geq (p(q) - c(\hat{q}))\,(1 - F(\hat{q}))\ \ \forall\ \hat{q} \neq q$$

Thus, for each type q, $c(q)$ must be chosen so as to maximize $u(q)$ with respect to q.

Note that the above story holds for the case of competitive supply. A monopolist might have an incentive to withhold capacity and charge higher prices. The monopolist can do so by offering another distribution of supply, G say, where $G \geq F$. It can be shown that, provided that marginal revenue is single-peaked, a monopolist will cut off the capacity at the monopoly supply, that is, at the point where marginal revenue equals zero.

5 Price dispersion

There is an important difference between price dispersion and price discrimination. Price dispersion results from imperfect information on the part of consumers, whereas price discrimination involves distinct prices for different types of consumer preferences. If all consumers would be perfectly informed, there would be no price dispersion, for all consumers would buy the good at the lowest price. Thus, different prices arise from different consumers having different information sets.

5.1 Search costs

If one assumes that acquiring information is costly to consumers, i.e., they have positive search costs, a reason for the differences in consumers' information sets may be differences in search costs. A consumer with low search costs will search longer, and may pay a lower price. However, a high search costs consumer may not find it worthwhile to continue searching, and pay the higher price.

The difficulty with this kind of models is what is known as the Diamond paradox. It can be illustrated as follows. Suppose that consumers all have a cost of search greater than some common lower bound γ, and that the search costs arise per store. It can be shown that in this model, all firms will charge the monopoly price, so the price dispersion disappears. The proof is by contradiction. Suppose that the lowest price is below the monopoly price, and consider a firm that sells at this price. It could raise its price by $\epsilon < \gamma$ without losing any customers, and thus increase profits. Therefore, a profit maximizing firm would never charge a price below the monopoly price.

5.2 Equilibrium price distributions

The old-fashioned way around this paradox was to assume differences between firms. However, this solution is not very attractive. The more modern models take another route. They assume that consumers have zero search costs or some other source of free information. By letting identical firms randomize, an equilibrium price distribution can be derived.

For example, consumers may get informative advertisements from sellers. Suppose n firms send advertisements, i.e., price offers, randomly assigned to a proportion α of the population. The consumers are passive; they buy at the lowest price offer they receive. First note that there is no equilibrium in pure strategies, that is, it cannot be an equilibrium for each firm to charge the same price. If this price gives all firms a positive profit, it would pay any single firm to undercut this price. If the price gives zero profit, any firm could earn strictly positive profits by offering a higher price, since there will be a fraction of the population that receives only his ad, and therefore buys at the higher price. So, in equilibrium, each firm will send out a random price offer.

Given that a consumer receives a firm's ad, and letting $F(p)$ be the probability that a price offer is not more than p, the firm's expected profit per consumer is

$$\pi(p) = (p - c)\,[1 - \alpha + \alpha(1 - F(p))]^{n-1}\,,$$

since a consumer will buy from the firm instead of one of its competitors if he does not receive an ad from a competitor or if the competitor's offered price is higher. It is also assumed that all consumers have the same maximum price for which they are willing to buy the good, v say. Note that there must be at least some firm that offers v. Suppose not. Then the firm that sends out the highest offer could have sent out a higher price, without losing customers. We can calculate the expected profit of offering v as $\pi(v) = (v - c)[1 - \alpha]^{n-1}$, since $F(v) = 1$. The last expression must be equal to $\pi(p)$ since all firms must have equal expected profits in equilibrium. By setting $\pi(p) = \pi(v)$, a closed form solution for the equilibrium price distribution can be derived. For higher α, the distribution puts more weight on prices close to marginal cost, and the expected value of the best price realized by consumers is closer to the competitive level.

6 The Coase conjecture

Selling a durable good creates a problem for a monopolist. In the first period, it is optimal for him to sell at the monopoly price. After selling the monopoly quantity, the monopolist is tempted to sell a bit more at a lower price to lower valuation consumers, and a rational monopolist will indeed do this. But rational consumers, who face the choice in which period to buy, correctly anticipate the monopolist's behavior. Some consumers will thus prefer to postpone their purchase to a later period, i.e., the monopolist faces a different demand. Whether or not a consumer will hold out for future prices depends on his valuation of the product over time and the discount factor. However, when the time period between two price changes becomes arbitrarily small, all consumers will wait and buy at a lower price. Furthermore, the consumers know that the monopolist has an incentive to keep the periods between two price changes small, since this increases his profits. Therefore, if the monopolist can change prices sufficiently fast, the price must go to marginal cost arbitrarily quickly. So, the monopolist will price at marginal cost and loses his monopoly power completely.

The problem for the monopolist is that he cannot commit to follow a predetermined sequence of prices, although he would like to. In other words, it is as if the monopolist competes with himself. It can be shown that if commitment were possible, the monopolist could just earn the static monopoly profit. Because consumers realize that committing to a price sequence is not subgame perfect, monopoly profits are reduced to zero.

Although is may be serious problem for the monopolist, one should not conclude that it is impossible for him to make any profits. There are several ways in which the Coase problem can be evaded.

One of the solutions is that the consumers' beliefs sustain a supergame equilibrium. The consumers believe that the monopolist will stick to the equilibrium price path, and if they observe an out-of-equilibrium price, they believe the monopolist to play the Coase path. This will cause prices to drop to marginal costs immediately. Since this causes profits to fall dramatically, this threat is sufficient to sustain the equilibrium. Other ways to circumvent the problem of being his own competitor in future periods are to lease the goods instead of selling them, to give a money back guarantee, or to reduce the durability of the good, i.e., to turn the good into a nondurable good.