Contests, Theory and Applications
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Abstract

This is a report on the course Contests, Theory and Applications by Kai Konrad at NAKE workshop at Vrije Universiteit Amsterdam 11-15 June 2001. In this report, I give a brief impression of this course and restructure the main contents as a clear summary. At the same time, I make some remarks with some more depth on the topics which interested me very much, especially for the problem of centest design. Moreover, I also express some individual viewpoints of mine in the application aspect of contest theory.

1 Introduction

As a standard part of our social, political and economic institutions, contests are ubiquitous in the modern economic world. There is not a standard definition of contests. However, we can try to understand it given the following three different definitions although they are not so precise. First, by Benny Moldovanu and Aner Sela (2000), contests are situations in which agents spend resources in order to win one or more prizes. A main feature is that, independently of success, all contestants bear some costs. Second, by Michael R. Baye, Dan Kovenock and Casper de Vries (1998), in the basic form of a contest, individuals submit entries (which may be bids, effort, or the commitment of other scarce resources which may be non-refundable) that influence the probability of winning some prize. Third, by Ani Dasgupta and Kofi O. Nti (1998), a contest is an economic

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or social interaction in which two or more players expend money or effort in hopes of winning a prize.

The prevalence of contests is obvious: employees spend effort in order to be promoted in organizational hierarchies, which often consist of several types of well-defined positions; dancers or singers compete for the first, second and third prizes in such competitions; homogenous product Bertrand competition is essentially a contest in which the contest entry is a price and the low-price firm wins the monopoly profits that correspond to the lowest price; in a first-or second-price auction, the prize is some object for which players' (private or common) valuations may or may not be affiliated, and the winning bidder pays either his own bid (in a first-price auction) or the bid of the second-highest bidder (in a second-price auction). There are more examples than I can put forward here.

On the other side, we should see the contest designers' goals. For instance, in a labor market tournament the goal of contest design might be to formulate contest rules that maximize total output or firm profits; the dance competition aims to get the overall high performance in order to thrill audience; market competition is for increasing the total social welfare[1]; the auction designer's goal is to maximize his total payoffs; furthermore, the teachers wish to maximize the expected learning effort made by their students so as to increase their study capabilities as well as knowledge level and so on[2].

Easily we can see that contest theory is such a theory to study the relevant problems with very broad scopes[3]. In this summary, apart from making a brief overview of contest theory including the its evolution, contest types, the main features, some other important related aspects of the theory and the applications in Economics, I will focus on two points of this course, some remarks about the problem of contest design and comments on the whole theory, especially for its application.

[1] It is an interesting question to probe into further. Here I just give a short explanation. For the firm competition in spite of the specific form it employs, we can take the market (the invisible hand) as the automatic contest designer or the government as the designer when this industry is under its control or it is influenced much by its regulatory reforms/policies. For the latter, it is obvious, the government's policy affects the firms behaviors and correspondingly determines the contest.

[2] Although on the surface, the objectives of the contest designers generally differ in different contest environments as above, we can conclude that all the contest designers' problems can be boiled down to how to allocate the prizes in order to maximize the sum of expected bids.

[3] Exactly speaking, contest theory is not a standard economic theory any more, it belongs to behavioral science.

2 Related Literature and Theory Evolution

2.1 An overview of theory development

Bertrand (1887) can be seen as the pioneer of contest theory although maybe he did not realize what he did would be related to the so-called contest theory. Francis Galton (1902) first posed the problem of prize allocation in contest. The most general form of contest originated in three separate literatures that emerged during 1960s and 70s: the auction literature pioneered by Vickrey (1961), the rent seeking literature pioneered by Tullock (1967, 1980), and the literature on animal conflicts pioneered by Maynard Smith (1974).

So, we can clarify the evolution process of contest theory:

Before 1950s, the beginning phase. Although some scholars put forward the related questions about contests, there was no unified theoretical framework about it and even no recognition that it could be treated as an independent theory;

1960s-1980s, the preliminary development phase. In this period, the basic concepts and the general framework emerged. It can be seen as the basis for further development;

The most developed phase of contest theory belongs to the last decade, especially the last five years since 1995. Many economists and much literature contributed to make it a mature topic. The theory got much extension and had been applied in many other fields. The booming phenomenon came into being. Of course, compared to the other economic theories, it is rather new with much more research still to be done.

2.2 The important literature

Here I can give some more detailed description about the evolution of the theory from the angle of addressing important literature as well as describing the main viewpoints. Continuing from above, Vickrey presents a formal game of private information in which bidders place refundable bids. The highest bidder receives the prize and pays either the highest bid (a first-price winner-pay auction) or the second highest bid (a second-price winner-pay auction). Tullock's (1967) treatment contains no formal model but describes an environment in which agents expend scarce resources in order to obtain transfers from other agents or, alternatively, to avoid having to give transfers to other agents. This environment is formalized in Tullock (1980), who presents a game in which players sink expenditures that influence their probability of winning a prize. Tullock employs a logit specification to describe a given player's probability of winning the prize: the probability of winning equals the player's expenditure raised to the power α divided by the sum of all

player's expenditure raised to the power α. A special case of Tullock model, that in which $\alpha = \infty$, shares a feature with the first-price winner-pay auction: the high bidder wins the prize and pays his bid. Unlike the Vickrey framework, however, losers also forfeit their bids. This special case of the Tullock model is, in fact, the first-price all-pay auction. The second-price all-pay auction originated with Maynard Smith's observation that an animal only has to fight until its rival gives up. Hence, the animal willing to fight the longest need only to expend effort up to the point where the next most persistent competitor gives up, which is why the second-price all-pay auction is more frequently referred to as the "war of attrition".

Some of the links between these three strands of literature have been examined by previous authors. Most notable among these is Hirshleifer and Riley (1978), and Riley and Robert (1990), which explore some of the connections between contests with refundable (auctions) and nonrefundable bids. Riley also made separate contributions to the theory of winner pay auctions (Maskin and Riley, 1992, 1993), second-price all-pay auctions (Nalebuff and Riley, 1985, and Riley, 1979, 1980), and first-price all-pay auctions (Hillman and Riley, 1989).

While winner-pay auctions are trivial to analyze when players enjoy complete information, the equilibria of first and second price all-pay auctions are more difficult to characterize because the resulting equilibria are in nondegenerate mixed strategies. Equilibria of the war of attrition are characterized by Bishop and Cannings (1978) and Hendricks, Weiss, and Wilson (1988), while characterizations of the first-price all-pay auction under complete information are provided by Hillman and Samet (1987), Hillman and Riley (1989), and Baye, Kovenock, and De Vries (1996). Moreover, these contests exhibit multiple equilibria which, for some parameter configurations, are not revenue equivalent (Baye, Kovenock, and de Vries, 1993, 1996).

For the case of incomplete information[4], the most general approach existing to date is based on Milgrom and Weber's (1982) seminal work on winner-pay auctions. Their framework postulates that agents receive signals of the value of the prize, and that these signals are affiliated and symmetrically distributed. This formulation permits agents' signals to be correlated, which turns out to circumvent the celebrated revenue equivalence theorem

[4]Normally, we can distinguish the contests as four types: Tullock-type contests, tournaments, first-prize-all-pay auctions and wars of attrition. To be simple, contests can be divided into two categories. One is the case that losers do not have to pay effort, e.g. first-price sealed bid auction. The other is that every contestant should pay effort, e.g. all-pay auction or a rent-seeking contest. Also we can employ some other ways for specific purposes. For example, like here, complete or incomplete information is another useful criterion.

that applies to symmetric equilibria in symmetric environments with independent signals. More recently, Krishna and Morgan (1995) use the Milgrom-Weber framework (along with some innovative distributional assumptions that guarantee the monotonicity of equilibrium bidding functions in the all-pay auction) to obtain a partial ranking of the revenues generated by the four mechanisms. As noted by Milgrom and Weber, when signals are affiliated the second-price winner-pay auction generates greater expected revenue than the first-price winner-pay auction and can not be uniformly ranked vis a vis the second-price winner-pay auction. Furthermore, they provide conditions under which the second-price all-pay auction generates higher expected revenue than the other three mechanisms.

In addition to these three main lines of research on contests, there is a diversity of literature that explores interesting field-specific applications of contests. There is also important work being done on the frontier of the field that extends contest theory to account for such things as risk aversion, budget constraints, threshold bid levels, the endogenous timing of moves, multidimensional bids, and dynamic (cumulative) bidding. Some of these topics will be surveyed in the following sections.

2.3 The latest research area

Nowadays, due to developments in contest theory, there are so many new fields being introduced into the eyes of contest theory as well as its application areas. But now I only give the most interesting topics that attract the economists and hence formed the main scope of modern contest theory. They are as follows:

First, contest design. It is a large topic which includes the structure of prizes or prize allocation[5], structuring noisy multi-stage contests and entry admittance. Rather than the contest models with complete information about the value of a unique prize (Tullock, 1980, Moulin, 1986, Snyder, 1989, Ellingsen, 1991, Baye et. al., 1993, 1996 and so on), more and more literature began to treat contests where multi-prizes are awarded so that it approaches much closer to real life. And also it is of interest to offer a rationale for both winner-take-all and multiple-prize contests in a single, integrated model. Broecker's (1990) model of credit markets has several features of an all-pay auction with as many prizes as contestants. Wilson (1979) and Anton and Yao (1992) study split award auctions that can be also interpreted as contests with several prizes. Barut and Kovenock (1998) study a complete information multi-prize contest with heterogenous prizes. Bulow and

[5]For my own interest, I will talk more about this topic. So the important viewpoints will be reviewed as the following. And also I will carry out some specific remarks on the topics of contest design and prize allocation in the next section.

Klemperer (1999) study an incomplete information model of a war of attrition with K identical prizes and $N + K$ contestants. Moldovanu and Sela's (2000) model is the latest development on this topic.

The second is strategic issues such as endogenous timing of moves and strategic delegation in contests. The related important literature is Dixit (1987), Baik and Shogren (1992), Baye and Shin (1999), and Konrad (2000).

Third, collective contests and the interaction between intra-group contest and between-groups contests. Lazear (1989) and Konrad (2000) discussed the problem of sabotage. The latter holds that due to a positive externality, sabotage is a "small number" phenomenon. Sabotage may increase lobbying efforts and the dissipation rate in lobbying contests, compared to a situation in which sabotage is not feasible. Two-stage collective rent seeking is another key aspect treated in Katz et. al. (1990), Nitzan (1991), Lee (1995) and Davis and Reilly (1999).

The fourth includes issues such as budget constraints, contribution caps, and iterated contests under such constraints. Among others, Che and Gale (1997, 1998), Glazer, and Konrad (1999) and Gavious et. al. (2000) are important literature. More specifically, Che and Gale (2000) describes the optimal mechanism for selling a good to a budget constrained buyer who is privately informed about her valuation and about her ability to pay. This mechanism involves non-trivial price discrimination (whereas it reduces to a take-it-or-leave-it offer if the budget constraint is known). Gavious et. al. (2000) show that for the contests where several privately informed agents bid for a prize, regardless of the number of bidders, if agents have linear or concave cost functions then setting a bid cap is not profitable for a designer who wishes to maximize the average bid. On the other hand, if agents have convex cost functions then effectively capping the bids is profitable for a designer facing a sufficiently large number of bidders.

3 Remarks about Contest Design and Prize Allocation

From what we discussed above, we can easily see that contest design lies in the heart of contest theory and its applications. For specificity, I take the paper of The Optimal Allocation of Prizes in Contests by Moldovanu, Benny, and Aner Sela (2000) as the example for analysis[6].

[6]It is an important paper in the field. So from the remark on it, we can get deep understanding of it and also with much generality.

In that paper, the authors study a contest with multiple (not necessarily equal) prizes. Ex-ante symmetric, risk-neutral contestants have independently distributed private information about an ability parameter c_i that affects their costs of bidding. The contestant with the highest bid wins the first prize, the contestant with the second highest bid wins the second prize, and so on until all the prizes are allocated. All contestants incur their respective costs of bidding. The contest's designer maximizes the expected sum of bids. Their main results are:

1) They display symmetric bidding equilibria for contestants with linear, convex or concave cost functions.

2) If the cost functions are linear or concave, then, it is optimal for the designer to allocate the entire prize sum to a single "first" prize.

3) The necessary and sufficient condition ensuring that several prizes are optimal is that contestants have convex cost functions.

4) Even if the designer can use instruments that exclude types with relatively low ability (whose increased bidding cause the benefit of having more prizes), the award of several prizes is advantageous.

From their model, I make some remarks which are related to contest design and also go beyond the topics of the above paper.

First, the relationship between the sum (value) of the prizes and the total expected bids.

We know for the problem of contest design, the starting point is the contest designer. And the most important thing is that the designer's task or goal focuses on the bids rather than "profit". Otherwise, the problem becomes very complex. Or we can say that the designer does not care about the value of prizes in such contests. We can even say, maybe more accurately, that the problem lies in how to allocate the prizes (or how to design the contest) while keeping the value of prizes fixed. Then there is an important assumption behind the model which is that there is no linkage between effort (bid) and the value of the prizes. In this way, the question is clear and specific and is also easy to solve. However, such a model can be seen as a static analysis because it neglects the choices or change possibilities for the aim of raising the total bids[7]. Given the real-life

[7]It is natural that most contest designers care about not only the total expected bids but also the values of the prizes. If some designer can provide prizes with cheaper prices, keeping the prizes structure fixed at the same time, and obtain the same or higher total expected bids, he must employ the new plan–everyone has the cost in the real world.

surroundings, the changes in the value of total prizes will affect the competition behaviors of contestants. The relationship between prize value and contestants bids is important to the designer's goal. So employing the dynamic method to analyze the prize allocation should be a relevant method. And then the concerning of profit is more reasonable.

Next, the definition of contest, mainly talking about the definition of bid.

Combined with that paper, we can take a rethinking of the definition of contest, more specifically, mainly about the implication of "bid" which is not defined so clearly in the related literature until now. According to the definition in that paper, contests are situations in which agents spend resources in order to win one or more prizes. A main feature is that, independently of success, all contestants bear some costs. To be more detailed according to this model, each contestant i submits a bid x_i (or undertakes an observable "effort") which causes a disutility (or cost) denoted by $c_i\gamma(x_i)$, where $\gamma : R_+ \to R_+$ is a strictly increasing function with $\gamma(0) = 0$, and where $c_i > 0$ is an ability parameter[8]. Note that a low c_i means that i has a high ability (i.e., low cost) and vice-versa. It is true under such framework there is no problem to carry out the analysis. But how to explain the "bid" exactly is still left unsolved. For example[9], what is the bid in some specific cases like in athletes competition[10]? More importantly, how to measure the bid? Such questions have not been answered very well so that the definition of contest is not so good. In their model, to some extent, they misunderstand the bids as only the entries; something like expenditure which incurs the cost. But to be accurate, and also from what the concept implies, bids should be the output or result of a contestant's expenditure. Apart from the feature of "observable", the bid should be "measurable" (or "evaluable" in some other cases[11]). Moreover, we should not mix the bid with effort together. In athletes competition, the effort can be speed, strength, technique and intelligence, even memory and so on which can be observable but difficult to measure. So sometimes, some contestant may pay much more effort but cannot increase their bids– maybe unsatisfying to the contestant. Also in athletes competition, the bids can be time,

[8] The main assumption here is the separability of ability and bid.

[9] Obviously, we can also put forward many other examples to illustrate this problem. But due to the paragraph and the logic behind this, it is unnessary to do so.

[10] It is addressed in the paper and the contestants' goals can be understood as competing for gold, silver and bronze medals.

[11] Take the example of Rubinstein international competition, young pianists compete for the first, second and third prizes. The bid in this case is not only the player's technique, but also the talent. And the feeling is important. So here "evaluable" should be emphasized.

height, length or distance and so forth. Hence, I do not agree with the explanation that a bid is similar to an observable "effort". The separability between these two concepts is necessary although indeed there is some relationship between them. Here I would like to address the paper Rent Seeking with Efforts and Bids by Haan and Schoonbeek (2001) in which the authors show us a good way to deal with such problems.

In a sentence, it is impossible to provide a unified standard of the definition of bid. We only can give a description about it and should make analysis according to specific cases. Of course, for most cases about economic contests, the features of "observable" and "measurable" are enough.

Thirdly, the intuitive explanation of the optimality of unique prize in the concave cost function cases.

The paper's explanation is as follows. Since a player with higher ability has a higher chance to win the first prize, increasing the value of the first prize by one penny causes an overall increase in equilibrium bids (and the increase is higher for higher abilities). Then for the contest in which the contestants are of concave cost functions, since the beneficial marginal effect of the second-prize on middle and low ability players is decreased, it is optimal to allocate the entire prize sum to a single first prize. This argument as well as the relevant mathematical proof seems reasonable. But it is not so clear or robust. Intuitively, will the middle and low ability players continue to attend such contests? Even if there is a strict asymmetric information assumption, the players know which level their own ability belongs to. So the sum of total expected bids in reality is ambiguous. Furthermore, in my opinion, to introduce experiment into such contest studies is useful.

The fourth, the lack of explanation to some more detailed questions.

The authors of the paper point out that for the case of convex cost functions, even if the designer can use instruments that exclude types with relatively low ability (whose increased bidding cause the benefit of having more prizes), the award of several prizes is advantageous. But there is no intuitional explanation to this result. I think the key reason lies in the fact that the remaining contestants staying in the contest still have convex cost functions.

Moreover, the paper does not mention the case of heterogeneity when there are some contestants with linear or concave cost functions and at the same time the other con-

testants with convex cost functions in some special contests[12]. In this case, I think, the optimal choice of the contest designer is to provide at least two prizes rather than just one. The intuition is simple: the unique prize design will cause the designer to lose the total bids of the middle and low ability contestants with convex cost functions. So the multi-prizes contest can give more incentives to that group.

4 Application in Industrial Organization and Other Comments

Contest theory can be applied into many fields such as labor economics, industrial organization, law and economics, political science, public economics and public choice and so forth. Baye, Kovenock and de Vries (1998) gave a good overview on it. Here I will emphasize its applications in the fields of industrial organization and transition economics.

From the angle of industrial organization, the firm competition for profit or market share and so on can be understood as the specific market contest in which the market (the invisible hand) is the automatic contest designer or we can take the government as the designer when this industry is under its regulation. Contest theory can be well applied in these fields. And especially for the latter, it is obvious that the government's policy affects the firms' behaviors and correspondingly determines the contest.

Furthermore, as for the transition economy (for example, in China), what is the optimal choice (i.e. the optimal contest design or prize allocation) during the course of regulation or deregulation reforms in the industries. The deregulation reform in banking industry since 1979 is a contest. And by empirical work (Ju and Yu, 1999), the welfare as well as the industry profit is increased. So does telecommunications in China. So we can say there is a close relationship between contest theory and industrial organization and transition economics. Of course, it is true to analyze the transition reform as a contest problem is a very big one. But it is an important development direction in its application.

And to some extent, contest theory generalizes winner-pay auction theory, it will also prove useful in the analysis of auction markets with all-pay components (see Baye, Kovenock and de Vries, 1991). Certainly many procurement contests take this flavor, involving multidimensional bids that have both winner-pay and all-pay components. The US Air Force procurement of an advanced tactical fighter (ATF) and Navy procurement of advanced attack aircraft took on characteristics of such a model. Firms had to cover

[12]It should be more close to real life because of the diversity of contestants in the same contest, which is worthy of some discussion.

substantial parts of the R&D cost of providing a prototype with no guarantees of recouping their investments. Many public and private sector procurement problems involve combination all-pay, winner-pay contests in which scarce professional time must be allocated in submitting prototype products or detailed description of how the procured task will be carried out. More generally, any theory of market interaction in which sunk selling costs influence the probability of a large sale can be analyzed through the use of multidimensional models of contests with all-pay and winner-pay components. In these markets, the contests framework permits one to model relationship-specific "scale effort" directly, and the framework can be extended with auction theoretic or bargaining theoretic models of price setting.

The other enlightenment is that we need to construct a new framework combining the contest theory with experimental economics. We can see from the existing literature, almost all the outcomes are based on paper work and strict assumptions. Although in most cases, they are right or reasonable, it is difficult to say whether it is true in the real world. Furthermore, the strong conditions for the theoretical environment restrict its applications. Just like what we talked about above, for the optimal prize allocation problem such as in the case of concave cost functions or in the case of both two different cost functions and so on, to analyze it by carring out experiments and to compare it with theoretical results will be better.

5 Conclusion

Although contest theory is not the standard economic theory, it has already gone beyond the original implication of contest itself and deals with more and more problems in very different areas as long as there is the environment like contest, it is a very strong analytical framework or tool for us to probe into the related topics with more depth. And with the development of modern economics, there are so many new fields which are suitable to this theory. So the future of its applications is quite bright.

In this report, apart from the short overview of the contest theory, I summarized its development process and the important research fields and key aspects of its application. From what I learned from the lectures, I also gave my own viewpoints on it, especially on the topics of contest design which interest me most. I think these remarks are important to compliment the theory. Furthermore I also put forward my opinions on the theory development as well as the new application areas. Of course, from this angle, we can say that contest theory is still a newly developing subject. Much more future work is needed.

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