Equilibrium Labor Market Flows
Dale T. Mortensen

Report by Thomas J. Dohmen, Maastricht University[¤]

1 Introduction

This is a report on the lectures on Equilibrium Labor Market Flows given by Dale T. Mortensen at the NAKE. Workshop in Amsterdam in December 1999. Dale Mortensen gave a good overview of the "°ow approach to labor markets", an important sub¯eld of labor economics. He considered a simple job search model to introduce the main concepts and tools of the °ow approach in some depth, which enabled him to cover extensions of the model in a brisk but clear way. These tools were subsequently applied in an equilibrium model of unemployment which is characterized by two sided search. The basic framework was extended to incorporate, inter alia, job creation and job destruction, to explain stylized facts on worker reallocation, and to analyze labor market policies. Finally a search model with wage posting was presented which can explain wage dispersion. In this report I follow a similar thread as in the lecture and choose to describe the basic framework in some detail while sketching possible extensions.

2 A Simple Model of Job Search

The key idea of equilibrium labor market °ow models is that labor markets are characterized by search and recruiting frictions. A job seeker is not informed about all existing job o®ers, but receives an o®er at random intervals. Upon arrival of an o®er, a decision has to be made whether to accept the job at the o®ered wage or reject it and search further. Similarly, ¯rms have to decide whether or not to post vacancies. In a simple version of a job search model, a number of simplifying assumptions are made.

[¤]Faculty of Economics and Business Administration, Maastricht University, P.O. Box 616, 6200 MD Maastricht. E-mail: T.Dohmen@mw.unimaas.nl

It is assumed that job offers arrive following a Poisson process with arrival rate λ in continuous time models, while job offers arrive with probability π in each period in discrete time models. This offer is a random draw from the wage offer distribution with distribution function $F(W)$. Job seekers know the offer distribution and hence the probability that the next job offer is more favorable than the current. While job seekers also know the rate at which a job offer arrives, they are uncertain about the exact timing and the wage, W, that will be associated with it. Next, it is assumed that job seekers enjoy some benefit b in each period of search, but incur a search cost c. Agents discount the future at rate r.

2.1 A Simple Stationary Job Search Model

Only transitions from employment to unemployment and vice versa are possible in a simple version, so that on-the-job search is precluded. Moreover, separations are final, i.e. there are no recalls. To greatly facilitate the mathematics, the simple model assumes stationarity, meaning that the parameters of the model (F, b, c, r, and λ or π) are constant over time. Unemployed individuals maximize the expected present value of their income over a finite horizon. Solving the model requires to determine an optimal stopping strategy, which maximizes the expected present value of acceptable employment net of the accumulated costs of search. An offer is acceptable, if and only if the wage W exceeds the value of the search option U_t, where U_t is given by

$$U_t = \frac{1}{1+r}\left[\begin{array}{l} b - c + (1-\pi)\,U_{t+1} \\ +\pi\int \max\langle \tilde{W}; U_{t+1}\rangle dF(\tilde{W}) \end{array}\right] \tag{1}$$

and $U_T = 0$ in the finite horizon case with discrete time. This can be solved by backward induction. In a discrete time stationary environment model, one can use the property that $\lim_{t\to\infty} U_t = U$.to obtain the infinite horizon solution for the reservation value U, which solves

$$U = \frac{b-c}{r+\pi} + \frac{\pi}{r+\pi}\int \max\langle \tilde{W}; U\rangle dF(\tilde{W}). \tag{2}$$

Similarly, one can solve for the reservation value in continuous time models. The assumption that offer arrivals follow a Poisson process with rate λ is useful because it conceptually connects the model with discrete time and the model with continuous time. The reservation value that follows from equation (1) approximates the continuous time solution with offer arrivals following a Poisson process when the time period becomes sufficiently small. In fact, the reservation value has to satisfy the following equation (in the limit)

$$rU = b - c + U + \lambda\int \max\langle \tilde{W} - U; 0\rangle dF(\tilde{W}). \tag{3}$$

in the finite horizon case. In the infinite horizon model a unique solution to the Bellman equation for U exists satisfying

$$U = \frac{b - c}{r + \lambda} + \frac{\lambda}{r + \lambda} \int \max\langle \tilde{W}; U \rangle dF(\tilde{W}). \tag{4}$$

The last two equations have the familiar structure of an asset pricing equation. The return to the "asset" U equals the sum of a utility flow in an interval, the expected capital gain in the interval and an option value. The option value arises from the fact that an offer can be rejected if it falls below the reservation wage. This essentially truncates the distribution of future income flows. The reservation wage, W^R, is the smallest acceptable wage offer which satisfies $W^R = rU$, such that the optimal strategy can be characterized by a reservation wage given by

$$W^R = b - c + \frac{\lambda}{r + \lambda} \int_{W^R}^{\bar{w}} \max\langle w - W^R \rangle dF(w) \tag{5}$$

The reservation wage rises with increases in b and λ, but is decreasing in c and r. Moreover, the larger the variance of the offer distribution, the larger will be W^R, as the option value increases. A job seeker becomes more likely to wait for another offer the larger is the probability that it is higher, and the larger is its expected value given that it is higher than the current offer.

The reservation wage does not depend on elapsed unemployment duration, a result of the stationarity assumption. Since the reservation wage is constant, the exit rate out of unemployment is constant, too. The exit rate θ is defined as the product of the job arrival rate and the conditional probability of accepting an offer which is in turn determined by the job offer distribution, i.e. the probability that an offer exceeds the reservation wage. But the job offer distribution is assumed to be constant as a result of stationarity and hence the hazard rate for the transition from unemployment to employment is constant.

This standard job search model can be extended to more realistically account for a number of features of labor markets. One could relax the stationarity assumption and allow some structural parameters to vary over time. The offer arrival rate may, for example, exhibit negative duration dependence because of a stigma effect on long-term unemployed. Alternatively parameters may change because of policy chances or business cycle effects. These are common examples of unanticipated nonstationarity.

2.2 Anticipated Nonstationarity

There are situations in which nonstationarity is anticipated. As an illustration, Mortensen analyzed the model when unemployment benefits are paid only for a finite period, as is the case in the U.S. Agents anticipate bene¯ts b to change. As a result, the reservation wage falls at the end of the bene¯t period. The solution proceeds by noting that the reservation wage is stationary after exhaustion of the bene¯t period. During the bene¯t period, the value of unemployment, and hence the reservation wage, is determined by an asset pricing equation. The value of unemployment U must fall, i.e. the "capital gain" must be negative as the return of the asset, i.e. the stream of bene¯ts, falls to zero at time T, because the exhaustion of bene¯ts is anticipated.

In models with anticipated nonstationarity, the optimal strategy of job seekers can be characterized by a reservation wage function $W^R(t) = rU(t)$ which is a di®erential equation of the form

$$\frac{dW^R(t)}{dt} = (r+\delta)\left[\begin{array}{c} rW^R(t) + \delta W^R(0) - (r+\delta)(b-c) \\ -\lambda \int_{W^R(t)}^{\bar{w}} [w - W^R(t)]dF(w) \end{array}\right] \qquad (6)$$

where δ is the separation rate. This di®erential equation has a unique solution, given some boundary condition, which derives from the assumption that the model is stationary for large enough t: In the example considered in the lectures, the model is stationary after the bene¯t period is exhausted at time T. We can determine the stationary value of the reservation wage W^{R*} for $t \geq T$. Using the boundary condition $W^{R*} = W^R(T)$, the di®erential equation (5) can be uniquely solved.

2.3 Other Extensions: Endogenous Search Intensity, Search on the Job and Job Shopping

The simple job search model can be extended to account for endogenous search intensity. This is readily done by making the o®er arrival rate and search costs a function of search intensity. One can assume, for example, that the arrival rate is proportional to search e®ort, s, i.e. $\lambda(s) = \lambda s$, while search costs are given by c(s) satisfying $c''(s) > 0$. Such a model implies that search intensity falls when bene¯ts rise.

Another important feature of labor markets are job-to-job transitions such that many new hires do not experience a spell of unemployment. This has motivated the development of repeated search models in which workers can search for better jobs after having accepted a job o®er. A simple model with on-the-job search assumes a stationary environment where employed search is similar to unemployed search.

An employed worker then receives an o®er at rate λ_1 while an unemployed job seeker receives an o®er at rate λ_0. If the two rates are the same, the reservation wage equals the bene¯t level. As soon as an o®er exceeds b, it is accepted because the probability of receiving a better offer in the future is not affected. However, if offers arrive at a lower rate when employed, the reservation wage is higher than b because it pays to wait as the probability of obtaining a better offer is higher when unemployed. The size of this mark-up on benefits is determined by the offer distribution and the difference between arrival rates.

The job separation rate exceeds the exogenous separation rate of the simple model with unemployed search only. The difference is due to the rate at which workers leave for other employers, which is equal to the product of the job offer arrival when employed and the probability that the wage exceeds the current wage.

Another extension of the model takes account of the phenomenon of "job shopping", which refers to the empirical fact that young workers experience many short job spells separated by unemployment spell. This can be modelled by assuming that the marginal product of an unexperienced worker in a given job is not known but that only the mean of the distribution is known. After a period of expected length $1/\eta$, the true productivity is observed and the worker separates if the value of unemployment is larger. Hence separation rates for inexperienced workers exceed those of experienced workers by η times the probability that the actual wage is smaller than the reservation wage.

3 Models with Equilibrium Unemployment

3.1 Two Sided Search

Mortensen introduced a search model with equilibrium unemployment, which works as follows. The economy consists of workers and employers. Workers seek jobs while employers can post vacancies. When a job-seeking worker meets an employer who has posted a vacancy, they decide whether to form a match that has some value X, which is drawn from a distribution F. To satisfy individual rationality, the worker and the firm only form the match if the share that each of them receives, $W(X)$ for the worker and $J(X)$ for the firm, exceeds the respective values of search, U for the worker and V for the firm. The value of search to the worker is described by equation (1). The value of search to the employer, V, solves

$$rV = \eta \left[\int \max\langle J(X); V \rangle dF(X) - V \right] - c; \qquad (7)$$

where c is the cost of posting a vacancy and η is the frequency with which an employer encounters workers seeking a job. Assuming that utility is transferable, such that $X = W(X) + J(X)$, a match will be formed iff the condition $X \geq U + V$ is satisfied. The reservation product R, which is defined as the minimum value of a match required to be accepted by the worker and the employer, is therefore given by $R = r(U + V)$.

The wage schedule $W(X)$ results from a rule to split the surplus that the match creates above the sum of the reservation values. It is often assumed that the wage schedule $W(X)$ is determined by a generalized Nash bargaining game, where U and V are the threat points for the worker and the firm respectively[1]. The resulting wage compensates workers for the forgone value of unemployment, rU, and pays in addition the fraction β of the surplus, where β is the (exogenous) bargaining power of workers. Since the wage rate, $w(x)$, is the money flow associated with employment, the value of employment $W(X)$ is given by $w(x) = rW$. Therefore we obtain

$$w(x) = rU + \beta[x - rU - rV]. \tag{8}$$

Other mechanisms for splitting the surplus can be formulated, which may give rise to other wage schedules. In fact, any division of the rents that satisfies individual rationality is possible. Alternative mechanisms include, for example, strategic bargaining games as described by Rubinstein (1982). The expected wage and hence the wage schedule is the same as in the generalized Nash bargaining game under the following rules: The worker is allowed to make a wage demand with probability β. After an offer is made the other party decides whether or not to accept. Given rejection the bargaining continues as long as the surplus remains positive, but both parties discount future returns at rate r. Alternatively, the same outcome obtains if negotiation stops in case of rejection and worker and employer both search for an alternative partner. However, other outcomes can result in general when the rules are changed, for example allowing for search during bargaining or introducing a breakdown possibility. Mortensen (1999a) discusses such issues in more detail and refers to the relevant literature.

3.1.1 The Matching Technology and Participation Assumptions

So far it has been described what happens when a worker encounters an employer with a vacancy, but it is not clear how and why they meet nor who searches. The matching technology and the participation assumptions are crucial elements of search models.

The matching technology relates the search and recruiting activities. Clearly, the flow rate at which unemployed workers meet vacancies must equal the rate at which employers with vacant jobs meet job-seeking workers. Since an individual unemployed

[1] See for example Pissarides (1990) and Mortensen and Pissarides (1994).

worker receives an offer at rate λ, the aggregate rate at which job-seeking workers meet vacancies is given by λu, where u equals the number of unemployed. If η is the rate at which an employer with a vacancy meets a job seeker and v represents the number of posted vacancies, then the matching technology, $M(v;u)$, satisfies by definition

$$\lambda u \equiv M(v;u) \equiv \eta v. \tag{9}$$

It is not sufficient to specify the matching function in an equilibrium model. A description of a search equilibrium requires the specification of u and v and the participation values of workers and employers, U and V. While the participation values U and V are generally determined as described above, different assumptions can be made about the specifics of unemployment and vacancy determination. One approach is to assume an unlimited number of workers and firms. In that case workers and employers enter until they are indifferent between participating and not participating. In the Pissarides (1990) model, introduced below, the supply of workers is inelastic while employers post vacancies until the value of a vacancy is driven to zero.

The participation assumptions and the matching technology have crucial consequences for the existence of a search equilibrium. The models discussed below generally assume that the matching technology, $M(v;u)$, is increasing, concave and homogeneous of degree one. But alternative specifications are possible. These include, for example, linear specifications, i.e. $M(v;u) = fu + gv$, linear homogeneous ($M(v;u) = \frac{(f+g)uv}{u+v}$) or quadratic ($M(v;u) = \left(\frac{f}{k} + \frac{g}{l}\right) vu$) specifications, where f and g are respectively the frequencies at which each unemployed worker calls a vacant job and vacancy posting employers call an unemployed worker. These different specifications result, as described in detail in Mortensen (1999a), from different informational assumptions. The linear model arises when each agent on one side of the market has a list of all unmatched agents on the other side of the market and calls one of these unmatched agents at a frequency of f and g.

So far, little has been said about job destruction. Implicitly it has been assumed in the discussion that jobs last forever. The model by Pissarides (1990) discussed in the next section assumes a constant exogenous rate of job destruction which results from a shock to productivity x. Job creation is endogenous in the model and determined by the participation assumption. Clearly, the probability that a match terminates is taken into account and changes the participation values U and V. Otherwise, the model works as outlined so far. A further extension of this model by Mortensen and Pissarides (1994) integrates job destruction by modelling the match product x as a stochastic jump process. This model will also considered.

3.2 The Pissarides (1990) Model

Pissarides specifies the match product x as a random variable that is realized at the beginning of the match and remains constant until the match resolves. Matches are destroyed due to idiosyncratic shocks that arrive at the constant exogenous rate δ. Given the destruction rate, the values of the match to worker and employer are given by

$$\begin{aligned} rW(x) &= w(x) + \delta[U - W(x)], \\ rJ(x) &= x - w(x) + \delta[V - W(x)]. \end{aligned} \tag{10}$$

Wages are determined by generalized Nash bargaining, which leads to a sharing rule as described in the previous section.

The process of job creation obeys the following rules. Employers post vacancies until all profit opportunities are exhausted, such that the value of a vacancy is zero in equilibrium. This requires that the expected cost of filling a vacancy equals the expected present value of hiring a worker. Participation of workers is determined by a fixed supply such that $u = 1 - n$, where n is the number of matches. The number of matches changes according to $\dot{n} = M(v, u)[1 - F(R)] - \delta n$. A new match is created if an employer with a vacancy meets an unemployed worker, which occurs at a rate determined by the matching technology $M(v, u)$, while at the same time the value of the match exceeds the reservation product, which happens with probability $[1 - F(R)]$. At the same time, matches are destructed at rate δ.

If the matching technology $M(v, u)$ is increasing, concave and homogeneous of degree one, a unique non-degenerate steady state search equilibrium exists. In that case the matching technology can be written as $M(v, u) = uM(\theta, 1) = m(\theta)u$, where $\theta = v/u$. θ is a measure of labor market tightness. The equilibrium unemployment rate equates the endogenous job creation rate with the constant job destruction rate and is therefore given by

$$u = \frac{\delta}{\delta + m(\theta)[1 - F(R)]}. \tag{11}$$

Consequently, the equilibrium wage is given by

$$w(x) = (1 - \beta)b + \beta(x + c\theta). \tag{12}$$

Obviously, the wage increases with the unemployment benefit b, with job productivity x, and with labor market tightness θ. An increase in cost of posting a vacancy reduces the value of search for an employer and hence increases the surplus of a match, i.e. results in a higher wage. An increase in benefits raises the reservation product, reduces tightness and therefore increases unemployment and the equilibrium wage. The effect of changes in F on unemployment is ambiguous as both the reservation product and labor market tightness increase when the variance of F increases.

3.3 A Model with Job Destruction: The Mortensen-Pissarides (1994) Model

Empirical evidence suggests that the rate of job destruction is not constant.[2] Mortensen and Pissarides (1994) take account of this empirical fact and make job destruction endogenous. They assume that the match product changes according to a stochastic jump process $x(t)$, where new values of x arrive according to a Poisson process with rate δ, where values of x are random with distribution function $F(x)$. These idiosyncratic shocks to the productivity of a given match affect the value of that match and hence the job destruction rate. A worker-employer pair separates when the value of the match falls below the reservation product R. Since a shock arrives at rate δ, the job destruction rate equals $\delta F(R)$, where $F(R)$ is the probability that the shock is sufficiently bad to drive the value of the match below the reservation product.

The reservation product depends on the values of the match for the worker and employer which are given by

$$\begin{aligned} rW(x) &= w(x) + \delta \int \left[\max\langle W(\tilde{x}), U\rangle - W(x)\right] dF(\tilde{x}); \\ rJ(x) &= x - w(x) + \delta \int \left[\max\langle J(\tilde{x}), V\rangle - J(x)\right] dF(\tilde{x}) \end{aligned} \tag{13}$$

The individual values of the match depend on the current state of match productivity, x, only. This is because productivity x evolves according to a stationary, persistent Markov process, so that it is sufficient to condition on the present state. Losses, arising when realizations of $\tilde{x}$, which represents all possible future states of productivity, are below the current match productivity, are truncated at $U - W(x)$ and $V - J(x)$ for workers and employers respectively. Workers always receive at least the returns from search, while employers can always obtain rV, by entering another business which has an initial productivity x_0. The model assumes that all jobs have the same initial productivity x_0.

Therefore the participation assumptions are virtually the same as in the Pissarides (1990) model. The free entry condition $V = 0$, must still hold. Moreover, since wages are renegotiated after each shock and are determined by generalized Nash bargaining, the wage rate is derived as $w(x) = (1-\beta)b + \beta(x + c\theta)$, just as in the Pissarides (1990) model.

[2] See for instance Davis et al. (1996).

In the steady state, the flow from employment to unemployment, i.e. $\delta F(R)(1-u)$, equals the flow from unemployment to employment, which is determined by the matching technology, $M(v;u)$. A non-degenerate steady state search equilibrium satisfies this steady state condition, the free entry condition and the reservation product equation. It can be shown that a unique equilibrium exists if $M(v;u)$ is increasing, concave and homogeneous of degree one.

The equilibrium wage and the equilibrium unemployment are then given by

$$u = \frac{\delta F(R)}{\delta F(R) + m(\theta)}$$

and

$$w(x) = R + \beta(x - R).$$

Since higher unemployment benefits, b, raise R and lower θ, the equations suggest that an increase in unemployment benefits results in a higher equilibrium wage and higher equilibrium unemployment. The new equilibrium comes about by a rise in the job destruction rate, $\delta F(R)$, and a lower job creation rate. The fall in θ suggests that the number of vacancies must fall.

4 Social Efficiency

The arrival rate of a job offer equals the number of matches per unemployed worker; i.e. $\lambda = \frac{M(v;u)}{u}$. Similarly the rate at which unemployed workers apply for a vacancy equals the ratio of encounters and vacancies, i.e. $\eta = \frac{M(v;u)}{v}$. Since the expected time until a vacancy is filled equals $1/\eta$, the participation decision creates two offsetting external effects. An additional vacancy reduces the rate at which vacancies are filled. But on the other hand it increases the rate at which unemployed workers receive an offer. Similarly, an additional unemployed worker imposes congestion effect on other job seekers, but positively affects the rate at which vacancies are filled. Given these externalities, it is not clear a priori whether the search equilibrium is socially efficient.

Social efficiency requires that the private returns to search equal the marginal social contribution of participation. It was shown in the lecture that social efficiency only obtains if $M(v;u)$ is homogenous of degree one and the surplus shares equal the elasticities of the matching function.[3] Hence a constant returns to search technology is necessary to ensure a social efficient equilibrium.

[3] Hosios (1990) shows this for a related class of models. Mortensen (1999a) contains a detailed analysis of the social planner's problem and derives the conditions for the matching function.

Alternative approaches to obtain social efficient outcomes are by Moen (1997) and Mortensen and Wright (1995). The basic idea behind these approach is that social inefficiency generally results because participants to search cannot trade future income after matching against the expected duration of the matching process. Moen (1997) solves this problem by changing the wage formation process. Mortensen and Wright (1995) obtain the same result by introducing a third party that offers searchers a pair of expected waiting time and expected income. If there is perfect competition among such middlemen, implicit prices are generated for expected waiting times which provide the correct incentives for participation that is socially optimal.

5 Fluctuations and Reallocation

The Mortensen-Pissarides (1994) model generates worker and job flows as a reallocation process from less productive to more productive jobs. In the version of the model as outlined above, job destruction results from idiosyncratic shocks. It is straightforward to include aggregate shocks as well by rewriting job productivity as px where x represents the idiosyncratic component that evolves as described above, while p is an aggregate measure describing the state of productivity. As p is the same for all agents in the economy at a given time, the equilibrium conditions can be rewritten as

$$\frac{cp\theta}{m(\theta)} = (1-\beta)\frac{cp\beta\theta}{r+\delta} \tag{14}$$

$$\left(R + \frac{\delta}{r+\delta}\int (x-R)dF(x)\right)p = rU. \tag{15}$$

Such a representation also lends itself for analyzing the implications of skill differences which can be interpreted as structural differences in productivity. Hence, in a model with two types of workers, high-skilled workers have a higher productivity index p than less-skilled workers. The equilibrium conditions imply that labor market tightness is higher for high-skilled workers and that the reservation product is lower for high-skilled workers. The latter conclusion might seem counterintuitive at first sight, but logically results from the fact that the returns to participation are higher for high-skilled workers.

The effects of cyclical fluctuations are assessed by assuming that p is the realization of a productivity shock. In a simple version, where $\{p(t)\}$ evolves as a Markov chain on $\{p_l; p_h\}$ with transition rate π_t, job creation rates and job destruction rates are negatively correlated. Mortensen and Pissarides (1999a) discuss such a model in detail. They devote special attention to job reallocation and employment fluctuation and the macroeconomic consequences for unemployment. In addition, they focus on how well the °ow approach, and in particular a model with productivity shocks, can explain the data.

The negative correlation is consistent with empirical ¯ndings. Mortensen explained the main ¯ndings of a detailed study of job and worker °ows in U.S. manufacturing by Davis et al. (1996). Some of the stylized facts, that a model of the labor market would have to be consistent with, include that job creation and destruction °ows are large and persistent in all industries and regions, that creation is procylical and destruction is counter cyclical - with a correlation between creation and destruction of -.36 for U.S. manufacturing. Moreover, destruction is more variable, idiosyncratic variation is large and worker °ows appear to be caused by job °ows.[4]

Cole and Rogerson (1996) have calibrated the Mortensen -Pissarides model and ¯nd that a model in which p is a stochastic process for an aggregate shock is broadly consistent with the time series features of job creation and job destruction.[5] Mortensen also showed how one can solve for the structural parameters of the model given the reduced form parameters. However, their parameters imply a low probability of ¯nding employment and hence a relatively high unemployment rate. The authors argue that this implies high "hidden unemployment", which shows up in non-participation in reality. However, the model does not account for °ows into employment from non-participants although this °ow roughly equals the unemployment-to-employment °ow. Moreover, the number of non-participants that reports a willingness to work is approximately as large as the number of unemployed.

6 Labor Market Policy

Mortensen explained how the Mortensen-Pissarides Model can be extended to assess active and passive labor market policies. In particular, the e®ects of unemployment insurance and social security ¯nanced by a payroll tax, ¿, levied on employers, as well as employment protection policies were analyzed. Firing restrictions can be modeled as a tax T that is paid when the match ends. Active policies like wage or employment subsidies can be

[4] Other real world features, that the model presented here cannot address, are that net changes increase with establishment size and decrease with age, while reallocation decreases with establishment size and age. These issues cannot be addressed because the model treats one employer-worker match as a ¯rm, such that ¯rm size is identical. The model does not consider job-to-job °ows either as it does not allow for employed search and therefore worker °ows equal job °ows, while in reality accession and separation rates are two to three times hire than creation and destruction rates. Such issues can however be incorporated in the search framework by extending the model in suitable ways.

[5] Mortensen and Pissarides (1999b) discuss the approach in detail in section 4.1.

modeled as a reduction in τ, while training or hiring subsidies can be modeled as a payment to the employer, or as a reduction in training costs C in models with training. Instead of deriving the results mathematically, which is done by Mortensen and Pissarides (1999a), I focus on the main implications and provide intuitive justifications for the results.

Since the payroll tax is levied on the employer, employers are concerned with $(1+\tau)w$ rather than w when calculating the value of the match. This reduces the match surplus and consequently lowers the wage, so that part of the tax is shifted to the worker. A firing cost reduces the employers' threat point in the wage bargain by T, which has a positive effect on wages. Higher unemployment benefits raise the search value U and hence wages.

An increase in unemployment income and higher payroll taxes both raise the reservation product, R, of matches and therefore increase the effective supply price of labor. This impacts on the match surplus and hence on job destruction. Job creation is also affected. The rate at which vacancies are posted decreases with a lower surplus, which results from a payroll tax and firing costs. Moreover, initial training costs affect the job creation decision by reducing the rate at which vacancies are posted.

A qualitative analysis of policy effects is conveniently done in the $R-\theta$ space. As job creation is given by

$$c\theta = m(\theta)(1-\beta)\left[1-\frac{\tau\beta}{1+\tau}\right]\left(\frac{x_0-R}{r+\delta}\right)-(T+C);$$

while job destruction is given by

$$\begin{aligned} & R+\frac{\delta}{r+\delta}\int_R^{x_u}(x-R)\,dF(x) \\ = \; & \frac{\tau\beta R}{1+\tau}+[1+(1-\beta)\tau] \\ & \times\left(\frac{\beta c}{1-\beta}\theta+b-\frac{rT}{1+\tau}\right); \end{aligned}$$

the job creation curve is downward sloping and the job destruction rate is upward sloping in the $R-\theta$ space. Since equilibrium unemployment is given by $u=\frac{\delta F(R)}{m(\theta)+\delta F(R)}$, higher values of R and lower values of θ are associated with higher equilibrium unemployment.

Hence, an increase in b only affects the job destruction curve. It lowers θ and increases R and therefore unambiguously increases equilibrium unemployment. An increase in the payroll tax shifts the job creation curve up and the job destruction curve down which reduces tightness while the effect on the reservation product depends on the relative slopes and the size of the shift. Hence the effect on unemployment is ambiguous. Similarly an increase in firing costs has ambiguous effects on unemployment. While the reservation product falls unambiguously, tightness may either rise or fall, depending on other parameters in the model. Finally, a rise in training costs reduces both tightness and the reservation product such that the effect on unemployment is again ambiguous.

Mortensen assessed the policy effects quantitatively in computational experiments where he assumed the same parameter of values as Cole and Rogerson (1996) in addition to setting $r = 0.01$ and $\beta = 0.5$, $M(v; u) = kvu/(v + u)$ and $F(x) \sim \text{uniform}(1 - \sigma; 1 + \sigma)$. These calculations suggest that an increase in the replacement rate ρ and an increase in the payroll tax τ have large effects on steady state employment and equilibrium wages in the absence of firing regulations. Moreover, the size of these effects is sufficiently large to explain differences in European and U.S. unemployment rates. However, the effects of a payroll tax and unemployment benefits on equilibrium income and employment are less clear in presence of firing regulation. In fact, steady state income can be raised above the no intervention level with moderate costs in terms of employment loss.

7 Wage Dispersion

One problem of the search models discussed so far is that they cannot explain wage dispersion. This has motivated a strand of the literature to develops models which generate wage dispersion. Simply assuming that employers set wages while job seekers either accept or reject has been shown not to work. In models with perfect information a Bertrand equilibrium results. Moreover Diamond (1971) found that in models with sequential unemployed search only, a unique equilibrium in which all employers post the same wage obtains even under imperfect information about offers. Hence different assumptions are necessary to generate wage dispersion.

Albrecht and Axell (1984) assume that agents have different costs of search; Burdett and Judd (1983) show that wage dispersion exists in models where workers receive more than one offer at a time. Finally Burdett and Mortensen (1998) generate wage dispersion in a model with on-the-job search. A similar model by Mortensen (1998) was explained in depth in the lectures.

The model allows workers to search when employed. The driving element of the model is the assumption that offer arrival rates are strictly positive, but that job offers arrive at different rates for employed and unemployed workers. The offer arrival rates for unemployed search, λ_0, and employed search, λ_1, are characterized by $\lambda_0 = m(\theta)$ and $\lambda_1 = sm(\theta)$. The properties of the equilibrium depend crucially on the value of s. As long as $0 < s < 1$, a unique dispersed equilibrium exists given that the matching technology satisfies certain assumptions. However, if $s = 0$, there is essentially no employed search and the model degenerates to a version of the Mortensen-Pissarides (1994) model; if $s = 1$, workers are indi®erent between employment and unemployment, such that all ¯rms o®er the reservation wage. Finally, van den Berg (1999) has shown that multiple equilibria can exist when $s > 1$.

In equilibrium, the unemployment rate balances the °ows into and out of unemployment. The wage distribution is then determined by equating the °ows out of and into employment at each wage o®er. It should be noted that the wage o®er distribution may di®er from the actual wage distribution. Wages and vacancies are determined by employers setting wages once and for all such that the value of the vacancy is maximized subject to the wage o®ers of all other employers. Di®erent wage o®ers can result which implies the possibility of dynamic monopsony. O®ering a higher wage attracts more workers and makes the current workforce less likely to be poached by other employers.

One implication of the model is that there is no symmetric wage equilibrium, because if everyone chose the same wage, an employer could deviate by o®ering a sightly higher wage to attract more workers. Therefore, o®ering a wage slightly above the equilibrium wage results in a discrete discontinuity in e®ective supply. As this is a contradiction, no symmetric equilibrium exists.

A discomforting feature of the equilibrium is that the wage distribution has an increasing density, while the opposite is observed in reality. A decreasing wage density can be generated by introducing productivity di®erences among employers. Such di®erences may stem, for example from heterogeneous production technologies. Examples of this approach are Mortensen (1990), Burdett and Mortensen (1998) and Bontemps et al (1999). These models have equilibria in which more productive employers o®er higher wages. Di®erences in productivity lead to non-degenerate o®er distributions. The wage distribution is contingent on the o®er distribution and search friction. The shape of the wage o®er distribution in turn depends on the productivity distribution. Consequently, a decreasing wage distribution results for particular shapes of the productivity distribution.

Intuitively these models work as follows. If ¯rms di®er in productivity, they will o®er di®erent wages in frictional labor markets. High productivity ¯rms likely o®er high wages as there is always a trade o® between the time it takes to ¯ll a vacancy (a®ecting the opportunity cost of no production) and the o®ered wage (a®ecting the ¯rm' share of output). Firms with a smaller marginal revenue product o®er lower wages, but can still ¯ll their vacancies because of the search frictions. Although their employees will accept higher wage o®ers from more productive ¯rms, i.e. will be poached away, there is also a steady in°ow as there are always unemployed workers who prefer a (temporary) job at the low-productivity ¯rm. Otherwise, the low-productivity ¯rm would not enter and post the vacancy in the ¯rst place. If there are su±ciently many ¯rms with below-than-average productivity, a skewed wage distribution as observed in reality will result.

8 Concluding remarks

Dale Mortensen has given a very good overview of the literature on equilibrium search models. Starting from a basic framework which introduces the main ingredients and tools, he has shown how the model can be extended to generate results consistent with empirical facts and how it can serve as a tool to analyze labor market policies. Just as the literature on the °ow approach matured over the past decade, the model presented in the lectures became more involved to ¯nally illustrate what happens on the frontier of present research on search models.

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