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ALTERNATIVE MODES OF ECONOMIC GOVERNANCE
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Handout 5

COLLECTIVE ACTION AND PUBLIC GOODS

It is sometimes argued that all problems of externalities and public goods are amenable to private solution because of "the Coase Theorem". Here we reexamine some aspects of this claim.

The Many Coase Theorems

Coase (1960)

When property rights are precisely defined and agreements costlessly enforced, two parties can efficiently internalize their externalities and reach a Pareto efficient outcome (a point in the core of their cooperative game).

Coase (1988)

The same with many parties. Also arguments that transaction costs will be efficiently economized, so outcome will always be constrained optimal.

Cheung (1970), Wittman (1989)

Governments and public policies will emerge when, and only when, they are more efficient than private arrangements. So whatever exists, whether a private agreement or a government for the provision of public goods, is for the best.

Counterarguments

Olson (1965)

General arguments that large numbers make efficiency unlikely, but no precise model.

Greenwald-Stiglitz (1986)

If transaction costs are due to information asymmetries, then the market equilibrium is not in general informationally constrained second-best.

Mailath-Postlewaite (1990) in the specific context of public goods.

Dixit-Olson (2000)

Most analyses assume that all the potential beneficiaries of a non-excludable public good are already identified and present to participate in the negotiation. But voluntary agreement should include the freedom not to participate at all. The game of negotiation should occur only among those who have chosen to participate.

1. VOLUNTARY PARTICIPATION

Non-excludable, discrete public good.
  Cost of production C
  Total population N; benefit to each V
  $M\ V > C > (M-1)\ V$, or
  $(M-1)/M < C/(MV) < 1$
  Relation to Palfrey-Rosenthal
Two stages
  1. Non-cooperative participation decision
  2. Cooperative provision decision among participants
Two versions
  1. Only one attempt to convene meeting
  2. Repeated attempts to convene meeting

Single attempt

Second stage: With n participants,
  go ahead if $n \geq M$; each pays C/n
First stage: Symmetric mixed strategy equilibrium
  P = probability of participation for each player
Equilibrium condition

$$\sum_{n=M}^{N} \binom{N-1}{n-1} P^{n-1}\ (1-P)^{(N-1)-(n-1)} \left[V - \frac{C}{n}\right]$$

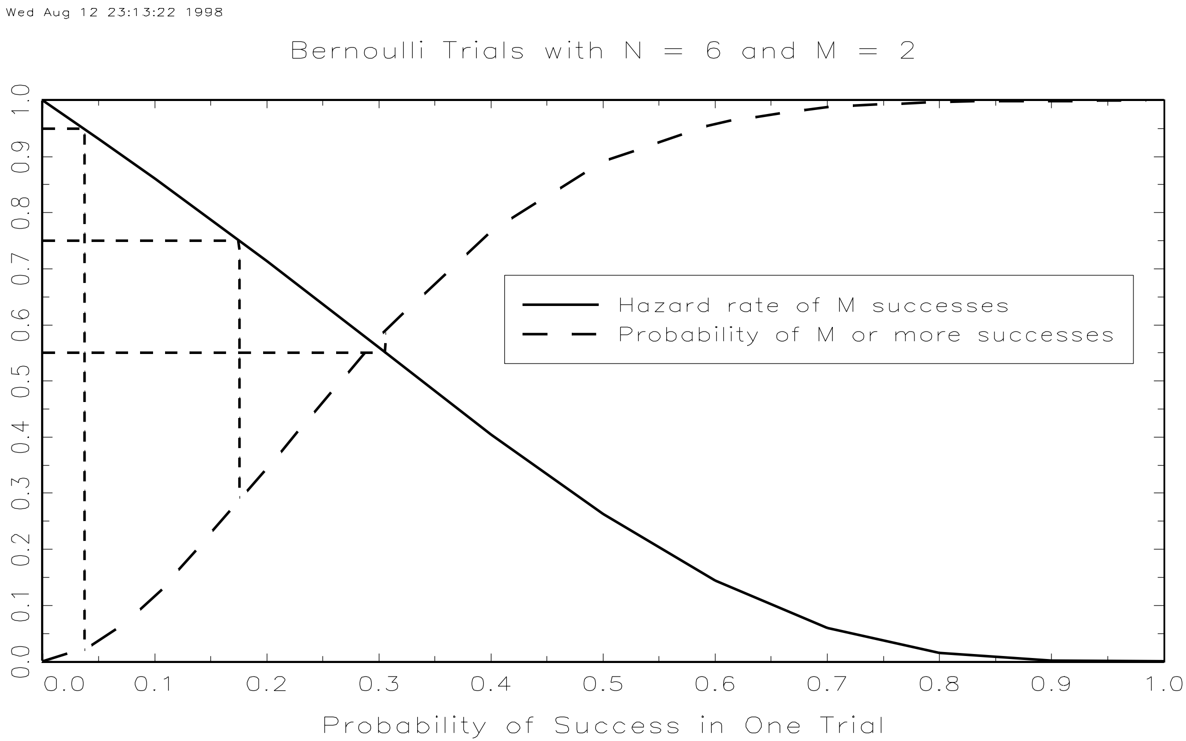
$$= \sum_{n=M}^{N-1} \binom{N-1}{n} P^{n}\ (1-P)^{N-1-n}\ V$$

Binomial probabilities

$$b(N, M, P) = \binom{N}{M} P^{M}\ (1-P)^{N-M}$$

Equation defining P

$$\frac{b(N, M, P)}{\sum_{n=M}^{N}\ b(N, n, P)} = \frac{C}{M\,V} \tag{6}$$


Wed Aug 12 23:13:22 1998
Bernoulli Trials with N = 6 and M = 2
Hazard rate of M successes
Probability of M or more successes
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
Probability of Success in One Trial

Numerical calculations of equilibrium

In all tables, $V = 1$.

Table 1 – $C = 9.5$, $M = 10$

N	Indiv. prob.	Cumul. prob
15	0.10	0.19 E -6
30	0.27 E -1	0.37 E -8
50	0.14 E -1	0.14 E -8
200	0.29 E -2	0.57 E -9

Table 2 – $C = 1.5$, $M = 2$

N	Indiv. prob.	Cumul. prob.
3	0.50	0.50
6	0.18	0.28
15	0.59 E -1	0.22
60	0.14 E -1	0.20

Table 3 –$C = 49.5$, $M = 50$

N	Indiv. prob.	Cumul. prob.
60	0.49 E -1	0.97 E -55
100	0.10 E -1	0.10 E -70
150	0.51 E -2	0.23 E -74
250	0.25 E -2	0.16 E -76

This mixed-strategy equilibrium can be "purified" in the usual way. Suppose each person's valuation of the public good is v, independently drawn from a distribution function $F(v)$ with support in $[\underline{v}, \overline{v}]$. Consider an equilibrium characterized by V, such that each person chooses to participate if and only if his privately known v exceeds V. But assume that when the players who choose to participate are assembled, they provide the good efficiently and share the cost equally. (Actually this may have its own information problems, which we examine later.) Then from each player's perspective, the probability that any one of the others will participate is $P = 1 - F(V)$. The equilibrium V must satisfy the same condition (zero net expected benefit) as before, and therefore (6) is still satisfied. But now V is endogenous. The left hand side of (6) is graphed as a function of P in Figure 1. As to the right hand side, as P increases from 0 to 1, V decreases from $\overline{v}$ to $\underline{v}$, so the right hand side is an increasing function. There is a unique equilibrium. If the uncertainty is small, so the support $[\underline{v}, \overline{v}]$ collapses on a point say V, then the right hand side becomes a horizontal line at an exogenous $C/(MV)$ and our earlier equilibrium reemerges.

Repeated attempts; $N = 2$

Discount factor β, close to 1
(Discounted present) value of game W
$N = 2$

An inefficient equilibrium

"Go-Ahead" expectations

Table 4: Payoff Matrix for Stage 1

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C,\, V - \frac{1}{2}C$	$V - C,\, V$
	OUT	$V,\, V - C$	$\beta W,\, \beta W$

Equilibrium conditions

$$W = P\,(V - \tfrac{1}{2}C) + (1 - P)\,(V - C) \tag{8}$$

$$W = P\,V + (1 - P)\,\beta W \tag{9}$$

An efficient equilibrium

"All-or-nothing" expectations

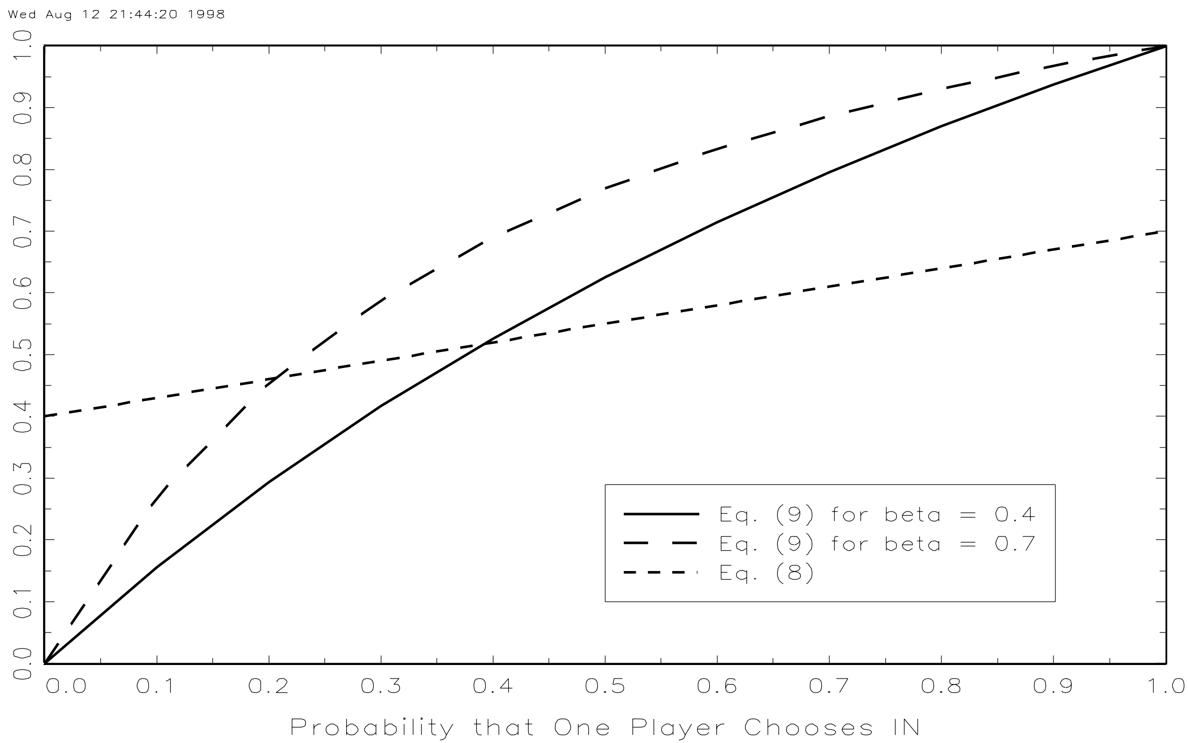
Table 5: Payoff Matrix for Stage 1

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C,\, V - \frac{1}{2}C$	$\beta W,\, \beta W$
	OUT	$\beta W,\, \beta W$	$\beta W,\, \beta W$

Incentive constraint:

$$V - C > \beta\,W = \beta\,(V - \tfrac{1}{2}C)$$

$$\beta > (V - C)/(V - \tfrac{1}{2}C)$$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0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Eq. (9) for beta = 0.4
Eq. (9) for beta = 0.7
Eq. (8)
Probability that One Player Chooses IN

Small transaction cost, $N = 2$

Cost of attending meeting ϵ
"Go-ahead" expectations

Table 6: Payoff Matrix for Stage 1

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C - \epsilon,\ V - \frac{1}{2}C - \epsilon$	$V - C - \epsilon,\ V$
	OUT	$V,\ V - C - \epsilon$	$\beta W,\ \beta W$

Equilibrium conditions

$$\begin{aligned} W &= P\,(V - \tfrac{1}{2}C - \epsilon) + (1 - P)\,(V - C - \epsilon) \\ W &= P\,V + (1 - P)\,\beta W \end{aligned}$$

"All-or-nothing" expectations

Table 7: Payoff Matrix for Stage 1

		Player B	
		IN	OUT
Player A	IN	$V - \frac{1}{2}C - \epsilon,\ V - \frac{1}{2}C - \epsilon$	$\beta W - \epsilon,\ \beta W$
	OUT	$\beta W,\ \beta W - \epsilon$	$\beta W,\ \beta W$

Equilibrium conditions

$$\begin{aligned} W &= P\,(V - \tfrac{1}{2}C - \epsilon) + (1 - P)\,(\beta W - \epsilon) \\ W &= P\,\beta W + (1 - P)\,\beta W = \beta W \end{aligned}$$

Solution

$$P = \epsilon\,/\,(V - \tfrac{1}{2}C)$$

2. INFORMATION ASYMMETRY

Even if all potentially interested players are assembled, there may be information asymmetry, which is a form of transaction cost. An example of such a model is Mailath and Postlewaite (REStudies 1990).

There are n potential beneficiaries of a discrete public good; the cost of providing it to n people is $C(n)$. The valuations v_i are independently drawn from distributions $F_i(v_i)$ with support $[\,\underline{v}_i, \overline{v}_i\,]$; the realizations are private information.

Mechanism design theory tells us that the optimal informationally feasible scheme can be thought of as a direct mechanism which asks individuals to report their valuations, and chooses the probability of providing the public good, say ρ, and the taxes levied on each individual, T_i, a functions of the reports of all, $(v_1, v_2, \ldots v_n)$. These functions should satisfy all the relevant individual rationality and incentive compatibility (truthful revelation) conditions, as well as budget balance.

Leaving out a lot of details, let

$$\rho_i(v_i) = \mathbf{E}_{-i}[\,\rho(v)\,], \quad t_i(v_i) = \mathbf{E}_{-i}[\,T_i(v)\,]$$

be person i's expected values of the probability of provision and tax payment respectively, where the expectation is taken over the distributions of the valuations of others, which are not known to i. Write the expected utility of person i knowing his true v_i as

$$U_i(v_i) = \max_{x_i} \,[\,\rho(x_i)\ v_i - t_i(x_i)\,]\,.$$

Then, for truthful revelation, we have the usual Mirrlees conditions: there are increasing function $\rho_i(v_i)$ such that

$$U_i'(v_i) = \rho(v_i), \quad U_i(\underline{v}_i) = 0 \quad U_i(v_i) = \int_{\underline{v}_i}^{v_i} \rho_i(z_i)\, dz_i\,.$$

The expected tax levied on person i is then $t_i(v_i) = \rho(v_i)\ v_i - U_i(v_i)$. Next we can compute the expected revenue, use integration by parts, and see that the expected budget constraint is met if

$$\int \cdots \int \left(\sum_{i=1}^{n} \left[v_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \right] - C(n) \right)\ \rho(v) \prod_{i=1}^{n} dF_i(v_i) \geq 0\,.$$

This is exactly as if everyone's willingness to pay is reduced by the inverse hazard rate $[1 - F_i(v_i)]/v_i$, a familiar result in mechanism design theory.

The true expected surplus generated by the mechanism is

$$\int \cdots \int \left(\sum_{i=1}^{n} [\, v_i - C(n) \,] \right) \rho(v) \prod_{i=1}^{n} dF_i(v_i) \geq 0\,.$$

The gap between the two creates the possibility of inefficient outcomes. Consider increasing the number of individuals, with independent identical valuation distributions for each. Mailath and Postlewaite prove that under mild conditions, as n goes to ∞, the probability that the public good *should* be provided goes to 1 while simultaneously the probability that it *is* provided in equilibrium goes to zero. The most important of these conditions are that there exist positive δ and ϵ such that, for all n,

$$\underline{v} + \epsilon < C(n)/n < \mathbf{E}[v] - \delta\,.$$

This requires the good to be desirable on average, but undesirable in the worse-case scenario, both by clear margins, no matter how small.