NAKE

Nieuws

volume 10, number 2

April 1998

Netwerk Algemene en Kwantitatieve Economie

Netherlands Network of Economics

NAKE Secretariat

Ms. Janna Mesker

Faculty of Economics,

University of Groningen

P.O. Box 800

9700 AV Groningen

Phone: 050-363 6664 / Fax: 050-363 7207.

E-mail: nake@eco.rug.nl

Home Page: http://www.eco.rug.nl/nake

PREFACE

It is running towards the end of April so we are getting ready for another NAKE workshop; in fact it is the twenty-fourth already. Four outstanding economists have agreed to come to the Netherlands and teach our students. Professor **Michael Whinston** (Harvard University but currently visiting Northwestern University) will be the microeconomic theory lecturer. His topic is broadly described as ‘Vertical Contracting.' Those of you who want a preview of what this is all about are referred to Whinston’s piece in the February 1998 issue of the *Journal of Political Economy*. Professor **Michael Woodford** (Princeton University) is a former classmate of Whinston’s at MIT and he will lecture on ‘Rules for Monetary Policy.' I must admit that I am somewhat tempted to send an invitation to Wim Duisenberg to attend the NAKE Workshop?! Professor **Tony Lancaster** (Brown University) is the author of the highly acclaimed Econometric Society Monograph entitled *The Econometric Analysis of Transition Data* (Cambridge University Press, 1990). The precise topic of his lectures is not yet known at this stage, but they are in the general area of ‘Microeconometrics.' Our fourth lecturer lives a little closer to us. Professor **Pierre Pestieau** (University of Liege) will talk on the highly policy-relevant topic of ‘Political Sustainability of Redistribution and the Reform of Social Security.' Overlooking this fine field of experts, I must admit that I am quite optimistic about the forthcoming workshop. For the second time in NAKE’s history, the workshop will be held at Wageningen Agricultural University.

ANNOUNCING THE NAKE DAY 1998

The (second) NAKE Day will be held on Friday, **October 23rd, 1998**, probably at the University of Amsterdam. The keynote speaker will be Professor **Eric van Damme** (Tilburg University) who will talk on a microeconomic topic. Up to date details concerning the NAKE Day can be found on the NAKE home page. Please reserve the date in your diary now. The NAKE Day is there for all Dutch economists, senior and junior alike. NAKE students who want to do their **final presentation** at the 1998 NAKE Day can send in their thesis proposal plus samples of finished work all throughout the year. See the NAKE homepage for details.

LAST BUT NOT LEAST...

In this *NAKE Nieuws* you find the best report on the third lecturer of last December's workshop in Nijmegen. **Charles Bos** (Erasmus University) gives his impressions on ‘Trends and Spurious Regressions' by Peter Phillips (Cowles Foundation & Yale University). Although these lectures were quite challenging, Charles has managed to give us a nice overview of the material that was covered.

FOR THOSE WHO MISSED IT LAST TIME AROUND

Please note the new address of the NAKE secretariat:

NAKE Secretariaat
Rijksuniversiteit Groningen
Faculteit der Economische Wetenchappen
T.a.v. Janna Mesker
Postbus 800
9700 AV Groningen
Tel: +31-50-363-6664
Fax: +31-50-363-7207
Email: nake@eco.rug.nl

Please share this information with any colleagues that may have dealings with NAKE.

COURSE QUESTIONNAIRE 1998/1999 UTRECHT PROGRAMME

In the middle section of this *NAKE Nieuws* you find a removable course questionnaire plus a listing of all courses that are offered by the Fellows of the network. **Note that one potential course was added compared to the previous listing**! Between 16 and 24 of these courses can be scheduled for inclusion in the Utrecht teaching programme for 1998-99. In order to design a well-balanced programme, it is important for me to know the potential clientele for the different courses. For this reason I would like to ask all (potential) participants to fill out the questionnaire and return it to the NAKE secretariat **before May 15, 1998**. The outlines of (almost) all courses can be found on the NAKE Home Page. Suggestions for new courses are, of course, also welcome. The new teaching programme for the academic year 1998-99 will be announced some time in June 1998 on the NAKE Home Page.

Ben Heijdra

NAKE WORKSHOP XXIV

8 - 12 June 1998

Wageningen Agricultural University

During the week from Monday, June 8th to Friday, June 12th, the Netherlands Network of Economics (NAKE) will organize a Ph.D. workshop. Four distinguished economists will teach intensive courses on microeconomics, macroeconomics, econometrics and public economics. Each course consists of five lectures spread out over five days.

Courses

Tony Lancaster, Brown University
‘Lectures on Econometrics'

Pierre Pestieau, University of Liege
‘Political Sustainability of Redistribution and the Reform of Social Security'

Michael Whinston, Northwestern University
‘Vertical Contracting'

Michael Woodford, Princeton University
‘Rules for Monetary Policy'

Register by filling out the form located in the middle of this *NAKE Nieuws* and returning it to the NAKE secretariat **AS SOON AS POSSIBLE**.

PROVISIONAL PROGRAMME NAKE WORKSHOP
WAGENINGEN, 8 - 12 JUNE 1998

Monday June 8	**Tuesday June 9**
09.30 - 10.30 registration/coffee 10.30 - 11.45 Lancaster 12.00 - 13.15 Woodford *13.15 - 14.15 Lunch* 14.15 - 15.30 Pestieau 15.45 - 17.00 Whinston *17.00 - 18.00 Welcome reception*	09.00 - 10.45 Woodford 11.15 - 13.00 Lancaster *13.00 - 14.15 Lunch* 14.15 - 16.00 Whinston 16.15 - 18.00 Pestieau
Wednesday June 10	**Thursday June 11**
09.00 - 10.30 Lancaster 10.45 - 12.15 Pestieau *12.15 - 13.30 Lunch* 13.30 - 15.00 Woodford 15.15 - 16.45 Whinston 16.45 - 18.15 Private consultations	09.00 - 10.45 Pestieau 11.15 - 13.00 Lancaster *13.00 - 14.15 Lunch* 14.15 - 16.00 Whinston 16.15 - 18.00 Woodford *20.00 workshop dinner*
Friday June 12	
09.00 - 10.30 Woodford 10.45 - 12.15 Lancaster *12.15 - 13.30 Lunch* 13.30 - 15.00 Whinston 15.15 - 16.45 Pestieau *16.45 - ... Closing drinks*	

REGISTRATION

Participation in the workshop is free for AIO’s/OIO’s of the institutions participating in NAKE, and includes tea, coffee, lunches, reception, as well as dinner on Thursday. The participants cover the costs of accommodation, breakfast, and the Course Readers. These costs, together with travel expenses, can however be declared at the faculties. Hotel rooms are available in the **WICC-hotel**. It is possible to share a room. Approximate prices are
ƒ 76,-- (single room) and ƒ 112,-- (double room).

The number of participants in the workshop is subject to an upper limit. NAKE students have precedence, and the date of receipt of the registration form is also taken into consideration. Since firm arrangements must be made for lunches, dinner, accommodation etcetera, we would like you to notify the NAKE secretariat in case of any alterations to your plans. Failure to do so may result in your being charged for the service you registered for. You register by filling out the form on the middle page (legibly and as completely as possible) and returning it to the NAKE secretariat **AS SOON AS POSSIBLE**. Upon registration you will receive written confirmation together with readers for the courses, hotel information, etc.

A number of AIO’s/OIO’s will be presented with the NAKE diploma during the workshop dinner on Thursday evening.

PRIVATE CONSULTATIONS

During the workshop it is possible for participants to have a one-to-one talk with one (or more) of the lecturers. Students who wish to make use of this opportunity are invited to hand in a brief (one-page) description of the research (-proposal) they would like to discuss. Each consultation will be approximately 30 minutes.

METHOD OF ASSESSMENT AND CREDITS

The NAKE workshops are mandatory for all first- and second-year graduate students following the NAKE programme. Hence, each student must attend at least four workshops. For three workshops the student must submit a written summary of the lectures of one course. This report must be based both on the notes taken during the workshop and on the assigned literature. These reports are assessed by the organiser(s) of the workshop. All (NAKE) students are expected to attend all sessions on offer during the workshop.

by means of the written report) is worth 2 "Study Points" (SP); 1 SP = 40 hours.

ADDRESSES AND INFORMATION

Location:	Wageningen International Conference Centre Lawickse Allee 11 6701 AN WAGENINGEN Room "Kleine zaal"
Information:	NAKE Secretary, Ms. Janna Mesker University of Groningen, Faculty of Economics P.O. Box 800, 9700 AV GRONINGEN Phone: 050 - 363 6664, Fax: 050 - 363 7207, E-mail: nake@eco.rug.nl Home page: http://www.eco.rug.nl/nake
Hotels:	WICC-hotel Lawickse Allee 11 6701 AN WAGENINGEN Phone: 0317 - 49 01 33 Fax: 0317 - 42 62 43 e-mail: hotel.congres@wicc-wir.nl
Local organizers:	Marrit van den Berg, e-mail: marrit.vandenberg@alg.oe.wau.nl Corjan Brink, e-mail: corjan.brink@alg.shhk.wau.nl Koos Gardebroek, e-mail: koos.gardebroek@alg.aae.wau.nl Erik Schmieman, e-mail: erik.schmieman@alg.shhk.wau.nl

Peter Phillips
Trends and Spurious Regressions

Report by Charles Bos, Erasmus University

1 Introduction

The subject of Phillips' lectures are the concepts of Trends and Spurious Regressions. As every dataset inherently consists of only a finite number of observations, finit sample theory will get some attention. However, more often the asymptotic or large sample theory will be used. This detour, from the finite sample to asymptotic theory to derive results that may be used in practical research, is made for two main reasons:

i) The spectral density and the autocovariance function contain an infinite number of parameters. Without asymptotics, there is no hope to be able to handle them in a reasonable way.

ii) Finite sample analysis is plain hard in the general time series context. Asymptotically, elements in the model may be disregarded as having no (asymptotic) effect, thus making asymptotic theory easier than finite sample theory.

Some subjects to be covered specifically are the concept of spurious regressions (Section 2), then the Signal-to-Noise Ratio in Section 3, the related Probability Excitation Condition which also relates to the possibility of trend extraction. In Section 5 regression with multiple signals has the focus of attention, followed by a short remark on Ito's calculus and a section on causality tests in VAR models (Sections 6–7). Stochastic nonstationarity and efficient least squares estimation are the subjects of the last sections.

2 Spurious regressions

The first publication on the concept of spurious regressions is probably the article by Yule (1926). He noticed that in some cases correlations between variables that had no logical connection are found. His famous example concerned the high correlation ($\rho = 0.95$) between the Church of England marriages and the mortality rate. Such relations are found more often. But even though the R^2 may be high, the relations usually do not stand up against a battery of other tests, like the Durbin-Watson. In general, the reliability of the relation for future forecast may be worse than bad.

Granger & Newbold showed, using a Monte Carlo sampling method, that such nonsense relations could easily be generated by relating two independent random walks. Take e.g. the following series:

$$X_t = X_{t-1} + u_t \tag{1}$$

$$Y_t = Y_{t-1} + v_t \tag{2}$$

where u_t and v_t are independent white noise processes. Then the relation $Y_t = \beta X_t + \epsilon_t$ generally results in a highly significant β. However, the residuals are non-stationary, and no predictive power is found in this relation.

The main advice of Granger & Newbold was to take first differences in the data, and to account for any further correlations by taking up lagged values of the variables into the regression. Although they do not stand on firm, theoretic grounds why such a procedure would resolve problems, for empirical work this solution can be satisfactory. This 'higgledy-piggledy' modelling, as it is keyed by Phillips, is not the final way to go. A better way is to use the 'general to specific' method of modelling: Start with a general framework, like a very long sequence of autocorrelations or an extremely densely calculated spectrum, and use those to find a more sparingly parametrized model that still captures the most relevant features of the data.

The theory on Function Spaces and continuous Wiener processes (so called after their inventor Wiener, who thought of them around 1930) are used in the article by Phillips (1986). With this theory, Phillips was able to derive asymptotic results that explained exactly the findings of Granger & Newbold. Another result of the influential article of Granger & Newbold is the exploration of the cointegrating relationships. Cointegration is a method to model long-lasting, non-spurious relations between non-stationary variables. Cointegration however is just a way of relating non-stationary variables. Some series, like income and consumption, are found to be co-moving rather than co-integrated. In many situations, straightforward detrending using a linear trend might be better than assuming unit-root stationarity and/or using a cointegration relation.

3 Signal to noise ratio

In engineering literature, the signal-to-noise ratio (SNR) is a common concept. It is used for getting an impression on the *possibility* of getting a good estimate of the parameters in a model. If the noise is very strong in comparison to the signal, estimation is hard in general. Phillips explained and derived the SNR in a couple of models. Write the general model as

$$y_t = s_t + u_t, \qquad t = 1, .., n \tag{3}$$

where y_t is the observation, written as the sum of the signal s_t and the noise u_t. The SNR is $\text{var } s_t / \text{ var } u_t$, where the variance of the signal is calculated over a sample of size n. Take as an example the linear trend model, $y_t = \theta t + \epsilon_t$. The variance of the signal $s_t = \theta t$ over the sample is

$$\begin{aligned}\text{var } s_t &= \theta^2 \text{var } t = \theta^2 \left(\frac{1}{n} \sum_{t=1}^{n} t^2 - \bar{t}^2 \right) = \theta^2 \left(\frac{1}{n} \frac{n(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \right) \\ &= \theta^2 \left(\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \right)\end{aligned}$$

Obviously, the variance of the noise is just σ^2, such that the SNR in this case is

$$SNR = \text{var } s_t \Big/ \text{var } u_t = \frac{\theta^2}{\sigma^2} \left(\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \right)$$

Phillips gave the SNR for several models as in Table 1.[1]. Note the huge difference in

Table 1: Signal-to-Noise ratios

Model	Signal s_t	Assumption	SNR
AR(1)	θy_{t-1}	$\theta < 1$	$\frac{\theta^2}{1-\theta^2}$
MA(1)	$\theta \epsilon_{t-1}$	$\theta < 1$	θ
Unit root	θy_{t-1}	$\theta = 1$	t
Regression	$x_t \beta$		$\frac{\beta^2}{\sigma^2} \frac{1}{n} \sum (x_t - \overline{x})^2$
Long memory regression	$(1-L)^d u_t$		$\frac{\Gamma(1-2d)}{\Gamma(1-d)^2}$
Linear trend	θt		$\frac{\theta^2}{\sigma^2} \left(\frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} \right)$
Reciprocal trend	$\theta \frac{1}{t}$		$\frac{\theta^2}{\sigma^2} \left(\frac{1}{n} \sum_{t=1}^{n} \frac{1}{t^2} - \left(\frac{1}{n} \sum_{t=1}^{n} \frac{1}{t} \right)^2 \right)$ $\rightarrow 0 \quad (n \rightarrow \infty)$
Power of time	$\theta \frac{1}{t^{\alpha}}$		$\frac{\theta^2}{\sigma^2} \left(\frac{n^{1-2\alpha}}{1-2\alpha} (1 - n^{2\alpha - 1}) - \frac{n^{-1-\alpha}}{(1-\alpha)^2} (1 - n^{\alpha - 1}) \right)$

signal strength between the AR(1) and the MA(1) model, especially if θ is close to one. The unit root case, which is the limit of $\theta \rightarrow 1$ of the AR(1) model, has a signal that

[1]Slightly adapted, to correct for an error

increases over time. In the regression case, β can only be estimated consistently (meaning that var $\hat{\beta} \to 0$ as $n \to \infty$) if $\sum x_t^2 \to \infty$ when n increases. The long memory regression has a strong signal if $d > \frac{1}{2}$. In the case of the reciprocal trend model, the signal is too weak: More data eventually does not lead to more information, with the SNR decreasing to zero. The regression model with a power of time as the regressand is the general case of the linear and reciprocal trend model. At $\alpha = \frac{1}{2}$ we find a border between sufficient and insufficient information in the regressand to be able to find a consistent estimate of θ.

4 The Probability Excitation Condition

Formally, the Probability Excitation Condition poses that the information content of the explanatory variables should be high enough to be able to obtain a consistent estimate of the variables. A powerful regressor can lead to a higher rate of convergence of the estimator. However, it might also lead to severe problems if the model is misspecified, or if the wrong regressors are used. Essentially, this is what is happening in the case of the spurious regression.

Take e.g. the linear trend model, $y_t = \beta t + u_t$. The signal strength depends mostly on the factor $\sum t^2 = O(n^3)$. To find a nondegenerate distribution of the OLS estimator of β, scaling by a factor of $n^{\frac{3}{2}}$ is needed:

$$n^{\frac{3}{2}}(\hat{\beta} - \beta) = \frac{\frac{1}{n^{\frac{3}{2}}} \sum t u_t}{\frac{1}{n^3} \sum t^2} \sim \frac{1}{\frac{1}{3}} \frac{1}{n^{\frac{3}{2}}} \sum t u_t$$
$$= \frac{3}{\sqrt{n}} \sum \left(\frac{t}{n}\right) u_t \sim 3\mathcal{N}\left(0, \sigma^2 \lim \frac{1}{n} \sum \left(\frac{t}{n}\right)^2\right) = 3\mathcal{N}(0, \sigma^2 \frac{1}{3}) = \mathcal{N}(0, 3\sigma^2)$$

Thus, the rate of convergence of the OLS estimator is $n^{\frac{3}{2}}$.

Similar derivations were shown for the standard stationary regressor case (leading to the standard rate of convergence of $n^{\frac{1}{2}}$, of course) and for a weak regressor: $x_t = 1 \Big/ \sqrt{(t)}$. This weak regressor was shown to fulfil the Probability Excitation condition, in the sense that eventually enough information was available to get a consistent estimate. The rate of convergence however was no more than $\log n$.

The data generated according to the reciprocal trend DGP did not contain enough information for a good estimate of β, we concluded from the SNR. Indeed, the error in the OLS estimate can be seen to be

$$\hat{\beta} - \beta \sim \frac{6}{\pi^2} \sum_{t=1}^{n} \frac{1}{t} u_t =: M_n$$

which is a martingale sequence when the u_t's are independent (but not necessarily normal). The second moment is

$$E(M_n^2) = \sum \frac{1}{t^2} \sigma^2 \to \frac{\pi^2}{6} \sigma^2 < \infty$$

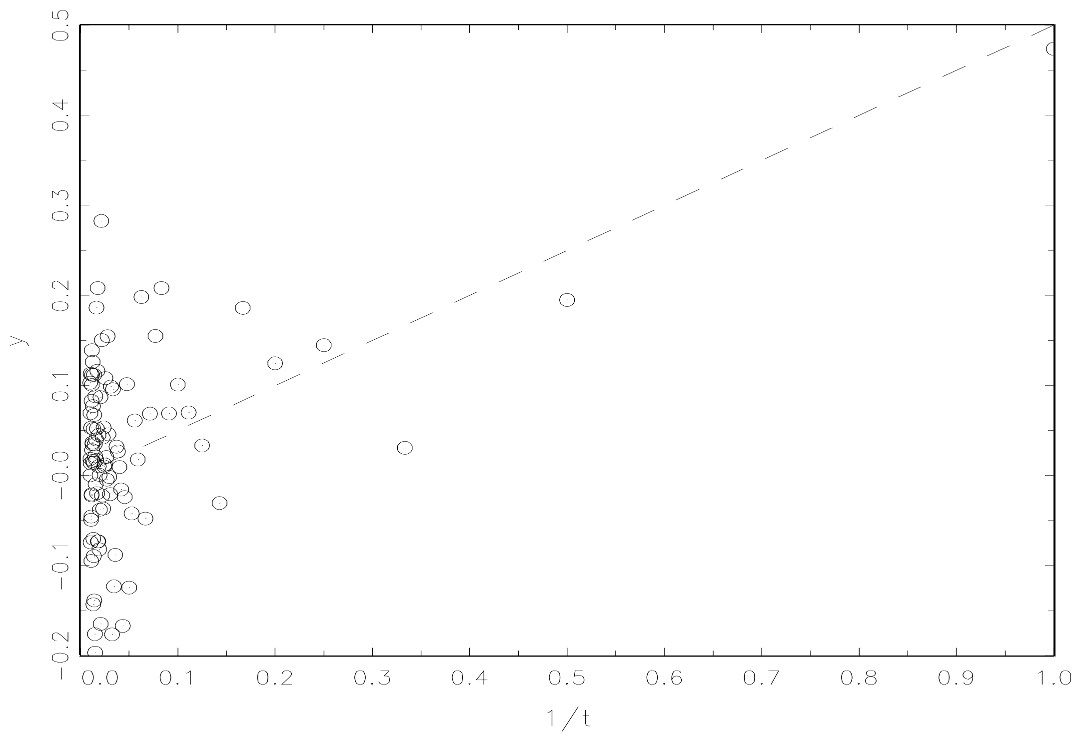

Figure 1: Weak identification in the reciprocal trend model

As the martingale is L_2 bounded, M_n converges almost surely:

$$\hat{\beta} - \beta \to \frac{\pi^2}{6} \sum_{t=1}^{\infty} \frac{1}{t} u_t$$

which is a random variable. Every simulation leads to a random outcome in the estimation of β, whatever the size of the sample is. In Figure 1 an example of a series generated according to the reciprocal trend DGP. It is clearly seen how more observations only lead to more 'noise' around the origin.

When investigating forecasts made with the models, one can make again the distinction into the several models.

1. Assume that the predictor P_t is a martingale. Then

$$\hat{P}_{t+k} = P_t$$

$$P_{t+k} - \hat{P}_{t+k} = \sum_{i=0}^{k-1} u_{t+i+1}$$

$$\text{var}\,(P_{t+k} - \hat{P}_{t+k}) = \sigma^2 k \to \infty.$$

   The forecast shows wider and wider confidence bounds as the horizon gets larger.

2. In the linear trend case, take as a prediction

$$\hat{P}_{t+k} = \hat{a} + \hat{b}(t+k)$$

which leads to

$$P_{t+k} - \hat{P}_{t+k} = (a - \hat{a}) + (b - \hat{b})(t+k) + u_{t+k}$$
$$\text{var}\,(P_{t+k} - \hat{P}_{t+k}) = \sigma^2 + \text{small}$$

where the equation for the variance is found after noticing that the estimation error in $\hat{a}$ and $\hat{b}$ evacuates to zero, at a rate of $n^{\frac{3}{2}}$. Thus, the confidence interval does not widen over a larger horizon in this case.

3. In the case of the stationary regression, we have

$$\hat{P}_{t+k} = \hat{\theta} P_t$$
$$\hat{P}_{t+k} - \hat{P}_{t+k} = (\theta^k - \hat{\theta}^k) P_t + \sum_{i=0}^{k-1} \theta^{k-1-i} u_{t+i+1}$$
$$\text{var}\,(P_{t+k} - \hat{P}_{t+k}) = \sum_{i=0}^{k-1} \theta^{2i} \sigma^2 \to \frac{\sigma^2}{1-\theta^2} = \text{var}\, P_t \quad \text{if } P_t \text{ is stationary}$$

Again, the prediction error interval takes up a fixed width, which is not really plausible in practice.

5 Regression with multiple signals

Take a model with multiple linearly trending regressors, as

$$y_t = \beta' x_t + u_t \qquad \mu'\mu = 1$$
$$x_t = \mu t + v_t \qquad \mu \in \Re^k$$
$$X = \tilde{t}\mu' + V \qquad \tilde{t}' = (1\ 2\ ..)$$

The information in the sample, $n^{-3}X'X = n^{-3}\sum t^2 \mu\mu' + o_p(1) \to \frac{1}{3}\mu\mu'$ has a singular limit, and thus the OLS estimate of β has a singular limiting distribution as well. Phillips demonstrated the use of a rotation $J = [\mu\ \mu_\perp]$. If the direction μ is assumed known, we see that the presence of strong information in one direction and little or none in other directions leads to different rates of convergence for different elements of β_r, the parameters in the model after applying the rotation. If β_r is re-transformed to the original coordinates, we find that the OLS results, though indicating a singular covariance matrix, are indeed correct: The information in the sample is such that only part of $\hat{\beta}$ can be consistently determined.

The message Phillips wants to convey with this asymptotic theory on multiple signals and different signal strengths is a very practical one: This theory implies that running

Vector AutoRegressions is not a bad thing to do. Even though the information content in the explanatory variables may be different, asymptotics still hold.

In case a unit root is present in the data, the standard results no longer apply. This is most easily seen in the AR(1) model $y_t = \theta y_{t-1} + u_t$, with the unit root as the limiting case.

$$\sqrt{n}(\hat{\theta} - \theta) = \frac{\frac{1}{\sqrt{n}} \sum y_{t-1} u_t}{\frac{1}{n} \sum y_{t-1}^2} \rightarrow \mathcal{N}(0, 1 - \theta^2)$$

When $\theta \rightarrow 1$, above result no longer holds, as the denominator is no long $O(1)$. In the stationary case, $\sum \theta^2 y_{t-1}^2 \sim O(n)$, whereas in the unit root case the same sum is $O(n^2)$. To capture this stronger persistence in the data, different scaling is needed:

$$n(\hat{\theta} - \theta) = \frac{\frac{1}{n} \sum y_{t-1} u_t}{\frac{1}{n^2} \sum y_{t-1}^2} \rightarrow \frac{\int_0^1 B(r)\, dB}{\int_0^1 B(r)^2\, dr}$$

The persistence here is so strong, that all of the trajectory of the process is needed in the analysis. The brownian motion $B(r)$ is used in this context. For more information on this distribution, see White (1958), Dickey & Fuller (1979) and Phillips (1987).

As the Bayesian statistics do not depend on asymptotic theory, i.e. on (Functional) Central Limit Theorems, their test statistics do not change. The main reason for this is that the Bayesians condition on the data, and thus the data is considered no longer to be random.

6 Detour: On Ito's calculus

Along with the theory on the asymptotic theory on the distribution of the OLS estimator in a unit root context, Ito's calculus is of much use. It is based on the first order derivative of the Brownian motion, $dB = \mathcal{N}(0, dr)$. From this we see that $E(dB)^2 = dr$ and that var $(dB)^2 = 2(dr)^2 = 0$. Thus, almost surely (a.s.) we have that $(dB)^2 = dr$. Note that in ordinary calculus $(dx)^2$ is negligibly small, whereas here the second term cannot be disregarded.

If we have a function $f(B)$ of the Brownian motion, then the derivative is

$$df(B) = f(B + dB) - f(B) = f'(B)dB + \frac{1}{2} f''(B)(dB)^2 + \text{small}$$

or, as it is more often used

$$df(B) \approx f'(B)dB + \frac{1}{2} f''(B)(dB)^2$$

7 Causality tests in general VAR's

The main idea behind Granger causality testing in a VAR context is to look at the covariance coefficients in the regression

$$y_t = A_t y_{1\,t-1} + .. + A_k y_{k\,t-1} + u_t = A y_{t-1} + u_t$$

with y_t the k-vector of the time series at time t, $y_{i\,t-1}$ the observation of time series i, lagged 1 period, and u_t a vector disturbance process.

$$A = \begin{pmatrix} a_{11} & a_{12}' \\ a_{12} & A_{22} \end{pmatrix}$$

To test for the causality relationship between y_t and $y_{2\,t-1}, ..y_{k\,t-1}$ the hypothesis H_0 : $a_{12} = 0$ is checked. If we write $a_{12} = S_{12}A$, with selection matrix S_{12}, the Wald test

$$W = \hat{a}_{12}' \left(S_{12} \hat{\sigma}^2 (X'X)^{-1} S_{12}' \right)^{-1} \hat{a}_{12}$$

If the data contains some unit roots, of which the directions are not known, then the OLS estimate of $\hat{a}_{12}$ has a singular normal distribution. The Wald statistic then has different convergence rates, corresponding to the different signal strengths. The final distribution of the Wald statistic is a linear combination between a χ^2 and a non-standard unit root test statistic. Toda & Phillips (1993) show that the classical tests are not asymptotically χ^2, and that they are dependent on unknown parameters. A general test in a classical setting on causality in a situation of VAR does not seem to be possible.

The standard way out is to 'misspecify' the model by increasing the degree of the autoregression. This way, all parameters may be root-n consistent, but the tests may have low power.

8 Stochastic nonstationarity

Theory up to this moment concerned stationary regressions, possibly augmented with (nonstationary) trends. In that case, standard asymptotic theory was still valid. For nonstationarity, regression has to be run in L_2, the space containing all square integrable functions:

$$L_2 = \{f | \int_0^1 f^2 < \infty\}$$

In this space, a series is decomposed into into its mean and the complement as

$$B(r) = \int_0^1 B(r) + (B(r) - \int_0^1 B)$$

When we have a Brownian motion $B(r)$, a constant and a trend can be fitted to it using $g = (1\ r)'$ by

$$\underline{B}(r) = B(r) - \left(\int_0^1 Bg'\right)\left(\int_0^1 gg'\right)^{-1} g(r) = \text{detrended } B(r)$$

As we can estimate a regression using prefiltering/detrending in Euclidian geometry, we can do the same in the Hilbert space:

$$\begin{aligned} y_t &= \alpha + \beta t + \theta y_{t-1} + u_t =: x_t + \theta y_{t-1} + u_t \\ \hat{\theta} - 1 &= y'_{-1} Q_x u \Big/ y'_{-1} Q_x y_{-1} \\ n(\hat{\theta} - 1) &= \frac{1}{n} y'_{-1} Q_x u \Big/ \frac{1}{n^2} y'_{-1} Q_x y_{-1} \to \int \underline{B} dB \Big/ \int \underline{B}^2 \end{aligned}$$

Every formulation of the regression equation has its own version of the regression function g to correct for it, thus resulting in a different asymptotic distribution.

9 Review of Efficient Least Squares

Back in the case of the linear regression, where $y = X\beta + u$ and $u \sim iid(0, \Omega)$, with $\Omega = 2\pi f_{uu}(0)$, we have the estimators

$$\begin{aligned} \text{OLS} : \hat{\beta} &\sim \mathcal{N}\left(\beta, (X'X)^{-1} X'\Omega X (X'X)^{-1}\right) \\ \text{GLS} : \hat{\beta} &\sim \mathcal{N}\left(\beta, (X'\Omega^{-1}X)^{-1}\right) \end{aligned}$$

From the efficiency analysis it is known that the variance of the OLS estimator is at least as big as the variance of the GLS estimator. But as the variance matrix Ω has $\frac{1}{2}n \times (n-1)$ parameters, we cannot estimate it on a sample of n observations. Also ML does not help, as even asymptotically there are more parameters than observations.

Two solutions are given in literature. First, Amemiya (1973) approximated the correlation structure in the errors u_t with an AR(m) model. For the asymptotic case, he had $m \to \infty$ but $\frac{m}{n} \to 0$. Then the resulting estimator can be estimated, and still has the GLS efficiency.

Hannan, in 1963, used an elegant solution. By using the Fourier transform of the observations into the frequency domain, in a first step the spectrum of the disturbances could be estimated consistently. With the results for this spectrum as weights, a weighted least squares led to an estimator of β with GLS properties.

In the fifties, the following theorem was found:

Theorem 9.1 *Grenander-Rosenblatt theorem:*
Trend extraction by LS delivers first order asymptotically efficient estimates, provided that $f_{uu}(\lambda)$ is continuous and $f_{uu}(\lambda) > 0, \forall \lambda$

Or, plainly speaking, stationary components do not matter when the β of a deterministic trend is estimated; it is found with GLS efficiency.

The conditions of the Grenander-Rosenblatt theorem are not fulfilled if at one frequency the spectrum has value 0. If $x_t = \epsilon_t - \epsilon_{t-1}$, the random walk as a regressor, then $f_{xx}(\lambda) = |1 - e^{i\lambda}|^2 f_{\epsilon\epsilon}(\lambda)$, which is zero for $\lambda = 0$. Thus, G-R does not work here. A second problem occurs in the case of long memory, where $x_t = (1-L)^{-d}\epsilon_t \to f_{xx}(\lambda) \sim \frac{f_{\epsilon\epsilon}(\lambda)}{\lambda^{2d}} \to \infty$ as $\lambda \downarrow 0$. Even though a long memory model may be stationary, G-R breaks down.

The extension to regression models with stochastic trends incorporated in the data is possible. If we e.g. have a model with a stochastic trend,

$$\begin{aligned} y_t &= \beta x_t + u_t \\ x_t &= x_{t-1} + v_t \end{aligned}$$

with u_t and v_t independent, then we can use the Functional Central Limit Theorem to find

$$\begin{pmatrix} \frac{1}{\sqrt{n}} x_{\lfloor nr \rfloor} \\ \frac{1}{\sqrt{n}} \sum_{t=1}^{n} u_t \end{pmatrix} \to \begin{pmatrix} B_x(r) \\ B_u(r) \end{pmatrix} = BM \begin{pmatrix} 2\pi f_{\nu\nu}(0) & 0 \\ 0 & 2\pi f_{uu}(0) \end{pmatrix}$$

The limiting distribution of the OLS estimator is found to be

$$n(\hat{\beta} - \beta) \to \left(\int_0^1 B_x B_x' \right)^{-1} \left(\int_0^1 B_x dB_u \right) \equiv MN \left(0, \omega_u^2 \left(\int_0^1 B_x B_x' \right)^{-1} \right)$$

with $\omega_u^2 = 2\pi f_{uu}(0)$ the long range variance of the disturbances u. In Phillips & Park (1988) the asymptotic equivalence of the GLS estimator to this OLS version is proved. It appears to be able to extent this result to the case where fractional processes are involved, though it has not been done yet.

10 Conclusions

In the last part of his last lecture, Phillips summarized some of the results of the previous days, and showed how they all interrelate. Especially in the field of cointegration, the results shown in these lectures lead to problems with modelling the lag structure, the number of cointegrating relationships, the estimation and the testing, in a consistent way. Work on the subject is underway, by Chao and Phillips. If they are able to convince their referees of the value of their solution in a Bayesian framework, it will be published soon.

REGISTRATION FORM

NAKE Workshop 8 - 12 June 1997, Wageningen Agricultural University

Name ....

Department ....

Institute ....

Address ....

Postal code/Place..

Phone: .... E-mail: ....

AIO/OIO/Otherwise..

(Please circle what is applicable)

Would like to have a private consultation with:

Lancaster, Pestieau, Whinston, Woodford

Will write a report on the lectures by:

Lancaster, Pestieau, Whinston, Woodford

Will attend the courses by:

Lancaster, Pestieau, Whinston, Woodford

Will attend:

reception (Monday) yes / no
workshop dinner (Thursday) yes / no
Special Dietary Requirements? yes / no
(give details if applicable)

Wants a hotel room for: Mon/Tues Tues/Wedn Wedn/Thurs Thurs/Fri

Will share a room: yes / no *(Single room ± ƒ 76,--*
double ± ƒ 112,--)

with..

Remarks:

Please return this form AS SOON AS POSSIBLE to the NAKE secretary.

QUESTIONNAIRE
UTRECHT-COURSES 1998/99

Please circle the number of the course(s) you wish to follow next year and return the form before **16 May 1998** to the NAKE Secretariat. In principle a course can only be scheduled once every two years. The courses marked with a star (*) have been given in the academic year 1997/98 and will therefore not **normally** be available in the academic year 1998/99. Courses in bold face are new.

10-week courses (4 SP = 160 hours)

	Teacher(s)	*Institute*	*Course*
1	van den Berg/van Ours/ den Butter	VU/KUB	Applied labour economics
2	van Damme/Peters/ Jansen	KUB/MU	Game theory
3*	Hartog/Teulings/Theeuwes	UVA	Advanced labor economics
4	Palm/Nijman	MU/KUB	Theory and application of modeling volatility in financial economics
5	Ridder/Wansbeek	VU/RUG	Econometrics of panel data
6	Talman/van der Laan	KUB/VU	General equilibrium model
7*	Folmer/de Zeeuw/	LUW/KUB	Environmental problems and policy: A) A theoretical introduction
	Withagen/Smulders	TUE/KUB	B) Growth and environment

5-week courses (2 SP = 80 hours)

8	Backhaus	MU	Recent developments in law and economics
9*	Beetsma	MU	Topics in international macroeconomics
10	van Bergeijk c.s.	DNB	Applied policy analysis
11	Bovenberg	KUB	Fiscal policy in open economies
12*	Brakman/Van Marrewijk	RUG/EUR	Regional economics, agglomeration and the global economy
13	Brenner	RUU	A critical view of economic theory
14	Brenner	RUU	Social economics: Heterodox approaches to economic theory
15	Broer	CPB/EUR	Dynamic general equilibrium modelling
16*	Burrell/Oskam	LUW	Agricultural policy analysis
17	den Butter	VU	Macroeconomic policy modelling
18	Cukierman	KUB	Central Bank strategy, credibility, and independence
19	van Damme/Gradus	KUB/MvFin	Topics in applied microeconomics: Deregulation
20	van Dijk/Boswijk	EUR/UVA	Econometric inference in dynamic models with integrated processes
21*	Ellman	UVA	The political economy of transition
22	**Ellman**	**UVA**	**The economics of famines**
23	van Ewijk/van Wijnbergen	UVA	Economic growth and development: Macroeconomics
24*	van Ewijk/Oosterbeek	UVA	Economics of education
25*	Furth/Van Cayseele	UVA/KL	Advanced industrial organisation
26	Garretsen/Sterken/Van Ees	KUN/RUG	Capital market imperfections, investment and monetary policy
27	de Gooijer/Franses	UVA/EUR	Recent Developments in Non-Linear Time Series Analysis
28*	Goyal/Janssen	EUR	Topics in advanced microeconomics
29	de Gijsel	MU	Micro-economische onderbouwing van een monetaire economie
30	Gunning/Keyzer	VU	Current issues in development economics
31*	Hartog/Theeuwes	UVA	Labour economics: A comparative empirical perspective
32	Heijdra/Meijdam	RUG/KUB	Intertemporal aspects of macroeconomics
33	Heijdra	RUG	New Keynesian macroeconomics

34	Heijdra	RUG	The macroeconomics of monopolistic competition
35*	Herings	KUB	Theory of incomplete markets
36	Hommes	UVA	Nonlinear economic dynamics
37*	Houba	VU	Differential games in economics
38	**Houba**	**VU**	**Strategic bargaining and endogenous threats**
39	Huizinga	KUB	International factor movements and international financial markets
40	**Jepma**	**RUG/UVA**	**International environmental policies**
41	de Jong, E.	KUN	Exchange rate determination and economic fundamentals
42*	de Jong, F.	KUB	Economics of foreign exchange markets
43	Keyzer	VU	Applied general equilibrium models
44	Kloek	EUR	Visualising data
45	Kooreman/Kapteyn	RUG/KUB	Intertemporal Choice
46*	Kooreman	RUG	The economics of household behaviour
47	van der Laan/Talman	VU/KUB	Economic equilibrium under price restrictions
48	**Magnus**	**KUB**	**Model selection and sensitivity**
49	**Magnus**	**KUB**	**Optimisation methods in econometrics**
50	Maks	MU	Competition and Market Coordination
51	Meijdam/Verbon	KUB	Theories of government debt
52*	Morgan	UVA	History of Economic Ideas
53	Morgan	UVA	History and philosophy of economic models
54	**Muysken**	**MU**	**Low skilled unemployment, job competition, and overeducation**
55	**Nijkamp**	**VU**	**Meta-analysis in economics**
56*	Peters/Storcken	MU	Social Choice Theory
57*	Pfann	MU	Optimal investment contingency plans of firms
58	**Potters**	**KUB**	**New institutional economics**
59	**Potters**	**KUB**	**Market micro-structure**
60	**Reuten**	**UVA**	**Heterodox economics: Marxian political economy**
61	**Ruys**	**KUB**	**Regulation and privatisation**
62	**Ruys**	**KUB**	**Competition and cooperation in the non-profit sector**
63	**Schoonbeek**	**RUG**	**Topic in oligopoly theory**
64	Schoorl	RUG	History of Dutch Economic Thought
65*	Schram/van Winden	UVA	Experimental economics and the design of mechanism
66*	van Soest/Melenberg	KUB	Applied non-parametric and semi-parametric econometrics
67	Steenge	UT	Rational choice theory and governance in the public sector
68	**Thijssen/Goudriaan**	**LUW/IOO**	**Efficiency and productivity analysis**
69	Uhlig	KUB	Business cycles
70	Verbon	KUB	Decision-making on intergenerational transfers
71	Vorst	EUR	Options pricing theory
72	Vorst/van de Sar	EUR	Behavioral Finance
73	de Vos	VU	Bayesian views on Testing and Model Selection
74	**de Vries**	**EUR**	**Risk management**
75	**de Vries**	**EUR**	**Advanced monetary economics**
76	**Wakker**	**LU/KUB**	**Prospect theory**
77	Wansbeek	RUG	Latent variables and methods of moments estimation
78	**Wansbeek**	**RUG**	**Econometric methods in marketing**
79	Weddepohl	UVA	Overlapping generations models
80	van Winden	UVA	Behavioural modelling of government decision-making
81	van Wijnbergen	UVA	Economics of transition
82	**Smulders**	**KUB**	**Endogenous growth theory**